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ABSTRACT

This memorandum discusses the theory of the determination of the apparent place of a star. The presentation identifies three fundamental effects that affect the numerical place of a star. First are the physical phenomena that affect the actual seeing of a star. Specifically, they are proper motion, parallax, and aberration. Second are the changes in the coordinate systems due to nutation and precession. Third is time which specifies when stars are seen and the epoch of coordinate systems.

This memorandum gives two methods for computing to within 5×10^{-8} radians the star's apparent place referred to the geocentric mean equator-equinox coordinate system of the nearest Besselian New Year. One method uses the star's reference apparent place in the "Apparent Place of Fundamental Stars"; the other uses the star's mean place of the same catalog. Both methods are equally accurate, but the method using mean place is preferred because it requires about one-half the input data of that using the apparent place.

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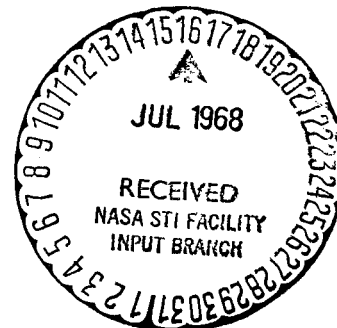(NASA CR OR TMX OR   MBER)   (CATEGORY)

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TECHNICAL MEMORANDUM

1.0 INTRODUCTION

In the Apollo program, an astronaut navigates in space by measuring the angle between a star and a planet or a landmark on the planet. A statistical navigation routine uses the difference between this measured angle and the expected angle to update the estimate of the spacecraft's position and velocity relative to some reference. To compute this expected angle, the astronaut must know the apparent place of the star and the planet; that is, he must know where they can be seen. Stars are also used to align the inertial platform to a known attitude so that thrusting can be measured.

This report shows how to determine the apparent places of stars from reference data. Specifically, the star's apparent place for an earth centered observer is computed, because navigation for Apollo is performed relative to the earth during a large portion of the mission. The star's apparent place is referred to the mean geocentric equator-equinox co-ordinate system of the nearest Besselian New Year because this is the standard inertial coordinate system used for Apollo. Two techniques for computing the apparent place to the same order of accuracy (5×10^{-8} radians) as the star data given in the "Apparent Places of Fundamental Stars" (APFS) reference are discussed. One uses the apparent star place data of this reference; the other uses the mean star place data of this reference.

The American Ephemeris (A.E.) is the standard star catalog for Apollo. Although the A.E. is not as accurate (5×10^{-6} radians) as the APFS, the A.E. is more than adequate because the sextant, the primary optical navigation instrument for Apollo, can only be positioned to within 10" (5×10^{-5} radians). The APFS is used for this memorandum for two reasons: First, it has two different kinds of star data which allows one

to compare and thus insure the correctness of the two computational techniques. Second, the effects of most all factors bearing on the apparent place determination of stars are evident because the APFS is sufficiently accurate.

The determination of the apparent place of a star is dependent on certain physical phenomena when the reference direction toward a star is specified at a time different from the time of observation and at a position different from the observer's. Section 2 of this report discusses these phenomena.

The mean place and the apparent place of a star given in APFS are both referred to certain coordinate systems. Section 3 of this report deals with these coordinate systems and the transformation required to go to the adopted inertial system. In particular, the term "mean equator-equinox" is defined.

Time is the parameter that specifies the occurrence of physical phenomena. Time must be specified both for apparent places and for coordinate systems which are dependent upon the dynamics of the solar system. Section 4 is devoted to this important topic. In this section, the Besselian New Year is defined.

Section 5 gives the specifics of the two methods for computing the apparent place of a star. A numerical example of each method is also given.

Section 6 gives conclusions based on the results given in Section 5.

2.0 APPARENT PLACE

The seeing of an object is dependent upon the light reflected or radiated by that object. Light travels outward from the object in a series of waves. In three dimensional space, these waves are surfaces of constant phase known as wave fronts. The locus of a point fixed on the surface of one of these traveling wave fronts describes a ray of light. If an observer is somewhere along this ray, then the direction opposite to the velocity vector of this point, when it impinges upon his eye, defines the instantaneous apparent direction toward the object to the observer. Astronomers call this the apparent place or position of the object. Because the star may move in the time between emission of light and perception of it, this direction is not geometric, but rather apparent, even if light is considered to travel in a straight line. As a matter of definition, the apparent direction toward a star from the sun is called the mean place of the star.

Three effects cause the observer to view the star differently from its mean place or, in other words, its reference (which is the sun unless otherwise stated) direction. First, the observer looks at it at some time other than the reference time. During this time span, there is a change in the star's apparent position; this change in the apparent direction to the star is called proper motion. Second, the observer is located elsewhere than at the reference; the change in the apparent direction toward the star due to the observer's position is called parallax. Third, the observer moves relative to the reference. The change in apparent direction toward the star due to the observer's velocity relative to the reference is called aberration.

All three of these effects are physical phenomena and are, therefore, invariant to coordinate transformations. For this reason, the discussion of coordinate systems is deferred to the next section. In this section, proper motion, parallax, and aberration are discussed, both separately and collectively.

2.1 Proper Motion

Proper motion is defined as the shift in the apparent direction toward a star in some time interval. The star's mean place given in APFS is given at a reference time t_o. At some other time t, the mean place is given by

$$\overline{P} = \overline{P}_o + (t-t_o) \, \overline{\mu} \qquad (1)$$

where $\overline{P}$ is the mean place at the desired time t (in years) and $\overline{P}_o$ is the mean place at time t_o. $\overline{\mu}$ is the annual proper motion of

the star. For an observer in the solar system, it is unnecessary
to correct for proper motion during the short time required by
the wave front that determines the star mean place to travel from
the sun to the observer (or vice versa). Light spans the solar
system in only some 12 hours; star motion during this time in-
terval is truly negligible.

2.2 Parallax

Parallax is the change in the apparent direction toward
a star from its reference direction caused by the difference in
position of the observer and of the reference. Formally, one
defines the parallax of a star (π, Figure 1) as the angle formed
in the plane containing the star C, the reference R, and the
observer O' when the distance RO' is one astronomical unit and
the angle RO'C is 90°.
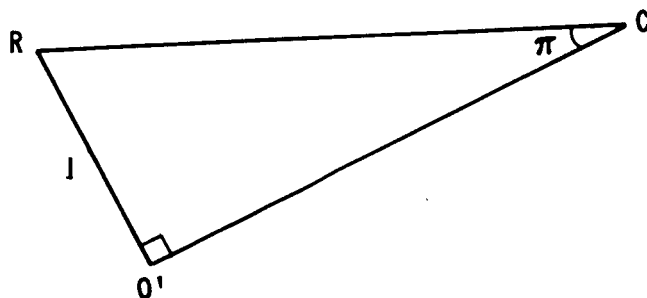
FIGURE 1

Then,

$$\sin \pi = \frac{1}{RC} \tag{2}$$

The observer is not generally in the plane where
parallax is defined. However, one can still determine the
apparent direction toward a star by vector addition if π is
known. In Figure 2, the observer may actually be at O. R,
O', and C are the same as in Figure 1. Call $\overline{RO} = \overline{R}$, and
$\overline{u}_{RC}$ a unit vector along $\overline{RC}$. (All distances are in astronomical
units)
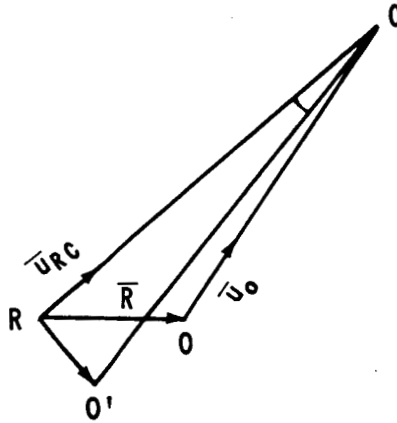
FIGURE 2

Then

$$|\overline{RC}|\bar{u}_{RC} = \overline{R} + \overline{OC} \qquad (3)$$

Solving for $\overline{OC}$ yields

$$\overline{OC} = |\overline{RC}|\bar{u}_{RC}-\overline{R} \qquad (4)$$

Factoring equation (4) by the magnitude of $\overline{RC}$ and substituting for 1/RC gives

$$\overline{OC} = (\bar{u}_{RC} - \sin\pi\overline{R}) \quad |\overline{RC}| \qquad (5)$$

Calling $\bar{u}_O$ the vector along $\overline{OC}$, equation (5) yields

$$\bar{u}_O = \text{unit } (\bar{u}_{RC} - \sin\pi\overline{R}) \qquad (6)$$

where $\bar{u}_{RC}$ is the unit vector of the star's reference direction specified at the same time as u_O (the difference in time interval for the wavefront to pass through the reference and the observer is ignored as is explained earlier).

Because star distances from the solar system are so large compared to the dimensions of the solar system, the approximation

$$\pi = \sin \pi \qquad (7)$$

is used, and equation (6) is then

$$\bar{u}_o = \text{unit} \ (\bar{u}_{RC} - \pi\bar{R}) \qquad (8)$$

where $\bar{R}$ is expressed in astronomical units, and π is expressed in radians.

2.3 Aberration

Aberration causes the apparent place of a star (or object) to be displaced forward of its reference position in the direction of the observer's motion. One can picture the effect of aberration by considering a point on a wave front of light traveling through a telescope until the point impinges upon an observer's eye. If the observer is at rest relative to the object, he can point his telescope (indicated by the dotted lines in Figures 3a, 3b, and 4) toward the object. The ray (indicated by the solid line in the same figures) described by the point on the wave front impinges upon the observer's eye as is shown in Figure 3a. However, if the observer moves with a certain velocity and continues to look in the same direction, the point on the wave front would impinge on the walls of the telescope rather than on the eye (Figure 3b).

0

(a)

0

(b)

FIGURE 3

It is then necessary to tilt the telescope forward so as to cause the point to travel down the centerline of the telescope (Figure 4).
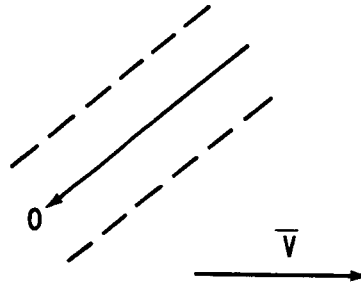
FIGURE 4

The amount of tilt required is, of course, dependent upon the observer's velocity.

The apparent direction toward a star can be found by vector addition of velocities if the observer and the reference occupy the same position and if the objection to the addition of velocities raised by relativity theory can be overcome. Appendix A shows that the difference between relativistic and non-relativistic aberration caused by the earth's orbital velocity about the sun is negligible, being only

$$\delta = -0.25 \times 10^{-8} \text{ radians}$$

while the star's place in the APFS is given to only 5×10^{-8} radians.

In Figure 5a, $\overline{V}_{RC}$ is the velocity of a point of light toward the reference. $\overline{V}_O$ is the velocity of the same point toward the observer. And $\overline{V}$ is the velocity of the observer relative to the reference.
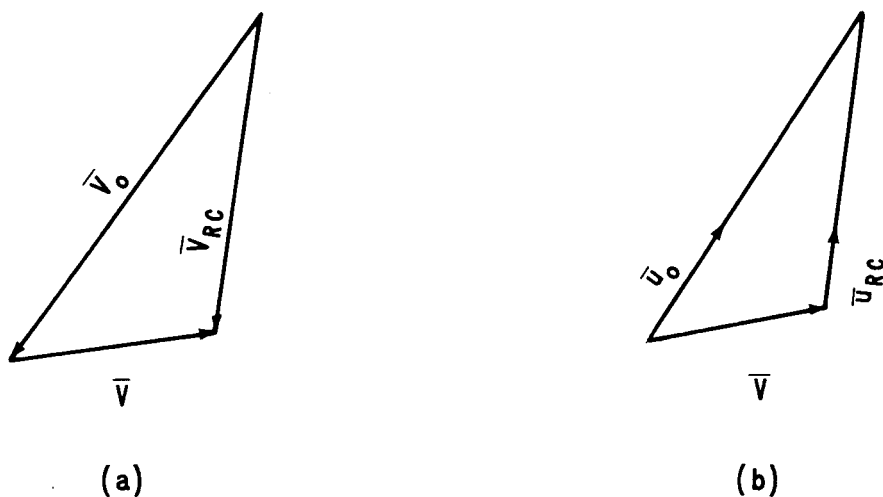
FIGURE 5

one has

$$\overline{V}_o = \overline{V}_{RC} - \overline{V} \qquad (9)$$

Apparent directions are defined by unit vectors $\overline{u}_o$ and $\overline{u}_{RC}$ (see Figure 5b) so

$$-V_o \overline{u}_o = -V_{RC} \overline{u}_{RC} - \overline{V} \qquad (10)$$

where $V_{RC} = c$, the speed of light. Substitution gives

$$-V_o \overline{u}_o = -c \overline{u}_{RC} - \overline{V} \qquad (11)$$

Divide through by c and unitize

$$\overline{u}_o = \text{unit } (\overline{u}_{RC} + \frac{\overline{V}}{c}) \qquad (12)$$

This is the desired expression for the star's apparent direction due to aberration.

2.4 Combined Proper Motion, Parallax, and Aberration

Generally, the effects of proper motion, parallax, and aberration are all present. Therefore an equation for the star's apparent place must include all three of these effects. To derive such an equation, one can be guided by the following three steps. First, one determines the point on the wave front that reaches the observer; this is found by first applying proper motion and then parallax to the star's reference direction. Finally, one determines the apparent direction by applying aberration.

Theoretically, whether the observer position (parallax) relative to the reference or the change (proper motion) in star apparent position from one epoch to another is accounted for first does not matter. Practically, because proper motion is specified by the same type of angles* as those expressing the star's mean place in the APFS one generally corrects for proper motion first. Consider $\overline{u}_{RC}$ as the reference direction corrected for proper motion. Then, the star's direction to a non-moving observer is given by equation (8) which applies parallax.

$$\overline{u}' = \text{unit } (\overline{u}_{RC} - \pi\overline{R}) \tag{13}$$

In equation (12) for aberration, $\overline{u}'$ replaces $\overline{u}_{RC}$ and

$$\overline{u}_a = \text{unit } (\overline{u}' + \frac{\overline{V}}{c}) \tag{14}$$

where $\overline{u}_a$ is the star's apparent place. Substitution of equation (13) into (14), gives

$$\overline{u}_a = \text{unit } \left[\text{unit } (\overline{u}_{RC} - \pi\overline{R}) + \frac{\overline{V}}{c} \right] \tag{15}$$

---

*Both the mean place of a star and its proper motion are specified by right ascension and declination (which are defined in the next section on coordinate systems).

For an observer in the solar system, the magnitude of $(\bar{u}_{RC}-\pi\bar{R})$ is so nearly equal to one that

$$\bar{u}_a = \text{unit}\left[\bar{u}_{RC}-\pi\bar{R}+ \frac{\bar{V}}{c}\right] \tag{16}$$

is valid with negligible error. Specifically, for α Centauri (maximum π), the nearest star, and an observer at Pluto (maximum R), the error does not exceed 1.1×10^{-8} radians.

The apparent place of a star from the earth is computed from a modified version of equation (16) because of the nature of all mean place catalogs of stars. Equation (16) requires both the position and the velocity of the observer relative to the reference. For earth - sun aberration, the need for velocity tables can be eliminated as is shown in Appendix B. This appendix shows that the earth's orbital velocity vector can be expressed by two constant magnitude components in the orbital plane. One component is perpendicular to the earth's position vector; the other is parallel to the semi-minor axis of the earth's orbit. By convention, the aberration caused by the latter velocity component is included in the star's mean place given in all mean place catalogs of stars, giving what is called the "catalog mean place." This included aberration expressed in angles is known as the E-terms of aberration.

A usable equation for the star's apparent place from the earth can be derived from equations in Appendix B. The velocity component parallel to the semi-minor axis is (equation B-13)

$$V_b^* = \frac{e\,\mu}{h} \tag{17}$$

and the velocity component perpendicular to the position vector, is (equation B-14)

$$V_f^* = \frac{\mu}{h} \tag{18}$$

where in both equations (17 and 18), μ is the gravitational constant, and h is the constant of angular mementum. In equation 17, e is the eccentricity of the earth's orbit. By vector addition,

$$\bar{V} = \bar{V}_f^* + \bar{V}_b^* \tag{19}$$

The catalog mean place ($\overline{u}_c$) is

$$\overline{u}_c = \text{unit} \ (\overline{u}_{RC} + \frac{\overline{V}_b^*}{c}) \qquad (20)$$

By applying equations (19) and (20), and neglecting the very small error (1.4×10^{-12} radians) caused by using $\overline{u}_c$ rather than the un-unitized sum of $\overline{u}_{RC}$ and $\frac{\overline{V}_b^*}{c}$, equation 16 becomes

$$\overline{u}_e = \text{unit} \left[\overline{u}_c - \pi\overline{R} + \frac{\overline{V}_f^*}{c} \right] \qquad (21)$$

where $\overline{u}_e$ is the apparent place of the star from the earth and $\overline{u}_c$ is the catalog mean place of the star given in the APFS corrected for proper motion. The stellar parallax (π) is obtained from the "General Catalog of Trigonometric Parallaxes", the proper motion is obtained from the "Smithsonian Astrophysical Observatory Star Catalog" (SAO), and $\overline{R}$, the earth's position, is obtained from the tables of the sun (which is equivalent to the earth's position by change of sign) given in the "American Ephemeris and Nautical Almanac." On the basis of the latest fundamental constants (see Appendix B, p. 7) adopted by the International Astronomical Union (IAU), the magnitude of $\frac{\overline{V}_f^*}{c}$, called the constant ($\kappa$) of aberration, is equal to 20".496; and the direction of $\frac{\overline{V}_f^*}{c}$ always leads the position vector $\overline{R}$ by 90° in the earth's orbital plane (ecliptic) looking down from the north.

For an Apollo mission, the apparent place of a star from the spacecraft can be found by

$$\overline{u}_a = \text{unit} \left[\overline{u}_c - \pi\overline{R} + \frac{1}{c} \left(\overline{V}_f^* + \overline{V}_{s/c} \right) \right] \qquad (22)$$

where $\overline{V}_{s/c}$ is the velocity of the spacecraft relative to the earth. The parallax due to the difference in position of the earth and the spacecraft is negligible. For α Centauri, the earth-moon parallax reaches only 9.9×10^{-9} radians as the spacecraft approaches the moon.

The use of equation (21) or (22) requires that all quantities be expressed in the same coordinate system. In the APFS, the star's mean place is expressed in mean equatorial coordinates. The earth's position $\overline{R}$, expressed in ecliptic coordinates, is chosen from the "American Ephemeris" because the direction of $\overline{V}_f^*$ relative to $\overline{R}$ is defined in ecliptic coordinates. One must then either transform $\overline{R}$ and $\overline{V}_f^*$ into the mean equatorial system, or the mean place into the ecliptic system. However, since the star's apparent place in the mean equatorial system is desired, one chooses to transform $\overline{R}$ and $\overline{V}_f^*$ into that system. These coordinate systems and the transformation between them are discussed in detail in the next section.

3.0 COORDINATE SYSTEMS

There are two coordinate systems that are generally used for astronomy, the equatorial and the ecliptic. The equatorial is based on the plane of the earth's equator, and the ecliptic is based on the plane of the earth's orbit around the sun. The X axis for both coordinate systems is the intersection of the two planes. The direction of the positive X axis from the earth is toward the point where the sun crosses the equator from south to north in its apparent orbit around the earth. This crossing occurs in the springtime. The positive X axis, so defined, is referred to as the "vernal equinox" or simply the "equinox". The projection of the X axis into the heavens occurs (or, at least used to occur) near the constellation of Aries ("the sun enters Aries"). For this reason, the equinox is denoted by the symbol for that constellation, Υ. The Y and Z axes of both coordinate systems complete a right-handed rectangular coordinate system. In the equatorial system, Y is positive 90° to the east of the vernal equinox along the plane of the equator; the Z axis is perpendicular to the equator, and is in the direction of the celestial north pole (the earth's north pole rotational axis). In the ecliptic system, the Y axis is positive 90° to the east along the plane of the ecliptic, and Z is perpendicular to the ecliptic so as to complete a right-handed coordinate system (see Figure 6).

One can transform from the ecliptic system $(X_{ecl}\ Y_{ecl}\ Z_{ecl})$ to the equatorial system $(X_{eq}\ Y_{eq}\ Z_{eq})$ by rotation about the X axis by an angle equal to the obliquity (ϵ). The obliquity is the angle between the ecliptic and equatorial planes. The rotation matrix is given by

	X_{ecl}	Y_{ecl}	Z_{ecl}
X_{eq}	1	0	0
Y_{eq}	0	$\cos \epsilon$	$-\sin \epsilon$
Z_{eq}	0	$\sin \epsilon$	$\cos \epsilon$

(23)

In both coordinate systems, the positions of a celestial body can be expressed in X, Y, and Z components or in angles. If angles (Fig. 7) are used, certain conventions are adopted. In the ecliptic system, angular positions are specified by longitude (λ) and latitude (β). Longitude is measured from the equinox positive eastward along the plane of the ecliptic. Latitude is measured from the ecliptic plane positive northward along a plane perpendicular to the ecliptic. In the equatorial system, angular positions are specified by right ascension (α), like longitude, and declination (δ), like
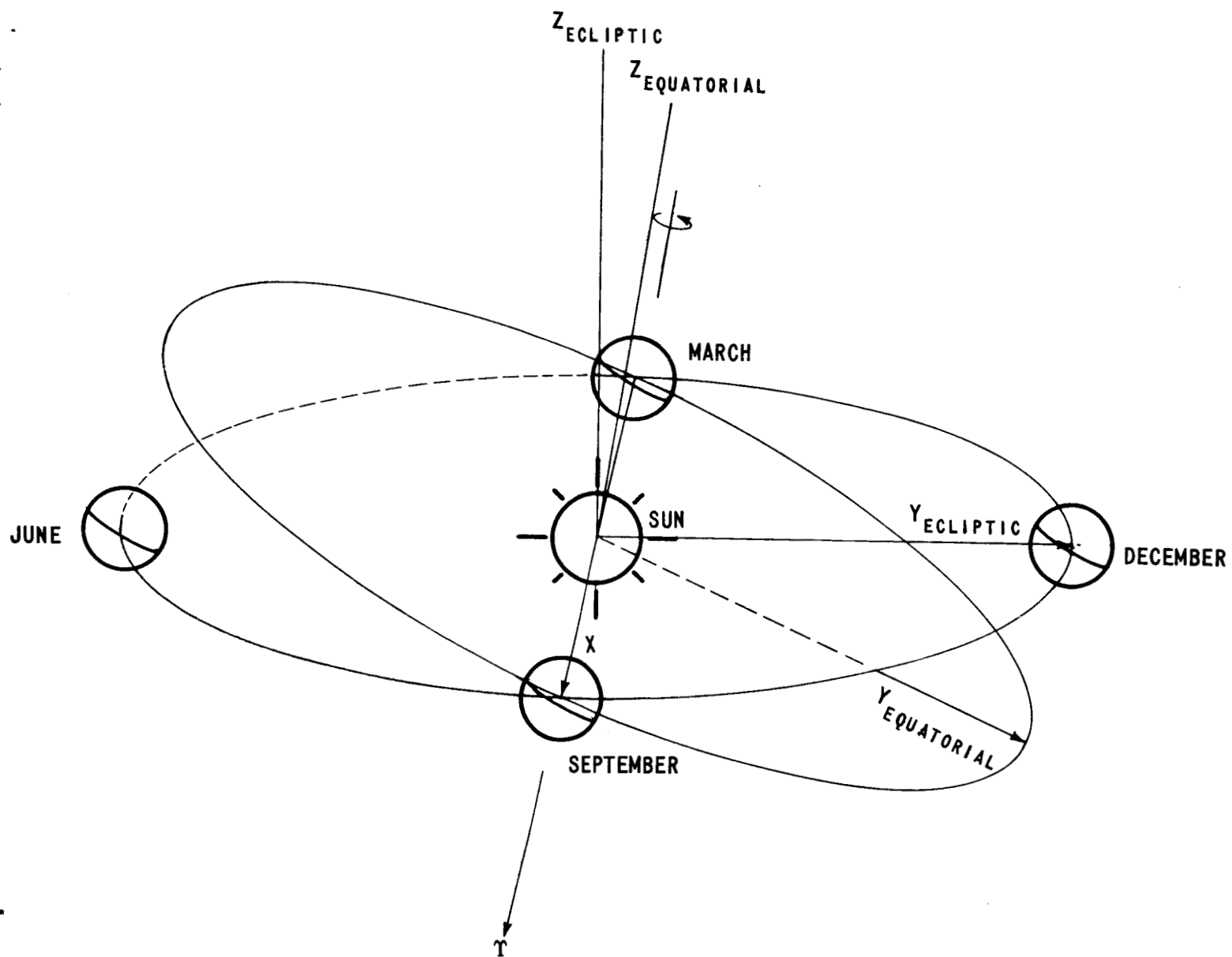
FIGURE 6 - EARTH'S ORBIT AROUND SUN

latitude, or by hour angle*(h) and declination. The right ascension is measured from the equinox positive to the east along the equatorial plane. The declination is measured from the equatorial plane positive to the north along a great circle (called a meridian) which defines a plane perpendicular to the equatorial plane. The hour angle is measured from a meridian positive westward along the equatorial plane.
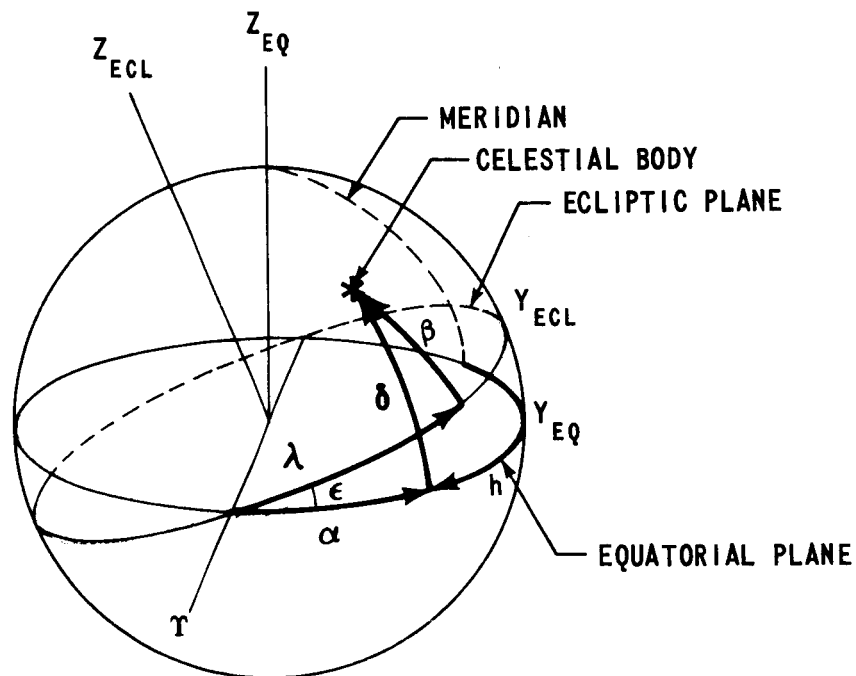
FIGURE 7

There are two types of equatorial coordinate systems: true and mean. The true equator-equinox coordinate system is based on the instantaneous orientation of the ecliptic and equatorial planes. Because of the gravitational pull of the sun and the moon, the equatorial plane wobbles irregularly with a short period; this is called nutation. If the effects of nutation are removed, one obtains the mean equator-equinox coordinate system. Gravitational forces also cause the mean equinox to rotate in a retrograde direction along the ecliptic, and the angle between ecliptic and equatorial planes to decrease. These

---

*24 hours, as an angle measure, always equals 360°.

very long period (when compared to that of nutation) effects are called <u>precession</u>. Because of precession, one must specify the epoch of the mean equatorial and ecliptic coordinate systems.

3.1 Nutation

Nutation is the short period irregular motion of the true pole around the mean pole. The angle between the poles is approximately 9", but this angle deviates somewhat from this value because of irregularities in the motion of the true pole. A complete rotation occurs in 18.6 years. However, the motion is more accurately represented by a series with components whose periods range from 18.6 years to just a few days. The nutation effects whose periods are more than 35 days are the "long period" nutation terms; the effects whose periods are less than 35 days are the "short period" nutation terms.

The total nutation (long and short period terms) is given by two angles. One is the difference ($\Delta\psi$) in longitude, measured along the ecliptic between the true and mean equatorial planes. The other is the difference ($\Delta\epsilon$) in the obliquity, of the true and mean equatorial planes. Values for $\Delta\psi$ and $\Delta\epsilon$ are tabulated daily in the "American Ephemeris and Nautical Almanac."

The short period terms of nutation, $d\psi$ and $d\epsilon$, are tabulated daily in the "Apparent Places of Fundamental Stars." To facilitate the interpolation in the ten day interval tables of star apparent places given in the APFS, the short period terms have there already been removed. That is, the star's apparent place, specified in angles of right ascension and declination in the APFS, is referred to the true equator-equinox coordinates of date, but with the omission of the short period terms of nutation. After interpolation, using second differences, these terms can be reinserted by differential correction (see page 157 of the "Explanatory Supplement") to the apparent place by

$$\Delta\alpha = d\alpha\,(\psi) \cdot d\psi + d\alpha(\epsilon)\, d\epsilon$$

$$\Delta\delta = d\delta\,(\psi) \cdot d\psi + d\delta(\epsilon)\, d\epsilon$$

(24)

where $d\alpha(\psi)$, etc. stand for partial derivatives $\frac{\partial\alpha}{\partial\psi}$ etc. They are as follows:

$$d\alpha(\psi) = \cos\epsilon + \sin\alpha\,\tan\delta\,\sin\epsilon = \frac{\partial\alpha}{\partial\psi}$$

$$d\alpha(\epsilon) = -\cos\alpha\,\tan\delta = \frac{\partial\delta}{\partial\epsilon}$$

$$d\delta(\psi) = \cos\alpha\,\sin\epsilon = \frac{\partial\delta}{\partial\epsilon}$$

$$d\delta(\epsilon) = \sin\alpha = \frac{\partial\delta}{\partial\epsilon}$$

(25)

$\Delta\alpha$ and $\Delta\delta$ are computed for the nearest integer dates preceding and following the desired time, and are then linearly interpolated to the desired time. The interpolated values of $\Delta\alpha$ and $\Delta\delta$ are added, respectively, to the already interpolated values of α and δ.

Values for $d\alpha(\psi)$, $d\alpha(\epsilon)$, $d\delta(\psi)$, and $d\alpha(\epsilon)$ are already computed and tabulated for each star in the APFS. The short period terms of nutation, $d\psi$ and $d\epsilon$, are tabulated daily in both the "American Ephemeris and Nautical Almanac" and the APFS.

Conversion from true equator-equinox coordinates (X_t, Y_t, Z_t) to mean equator-equinox coordinates (X_m, Y_m, Z_m) can be accomplished by the following transformation matrix:

	X_t	Y_t	Z_t
X_m	1	$\Delta\psi\,\cos\epsilon_o$	$\Delta\psi\,\sin\epsilon_o$
Y_m	$-\Delta\psi\,\cos\epsilon_o$	1	$\Delta\epsilon$
Z_m	$-\Delta\psi\,\sin\epsilon_o$	$\Delta\epsilon$	1

(26)

where ϵ_o is the mean obliquity. The matrix just given is an accurate approximation of the exact transformation as is shown in Appendix C.

3.2 Precession

The smooth retrograde (clockwise, looking down on the plane of the equator from the north) rotation of the equinox in the ecliptic plane with a period of about 26,000 years (see Figure 8) is also caused by the gravitational forces of the sun and the moon on the earth's equatorial bulge plane. This is called luni-solar precession.
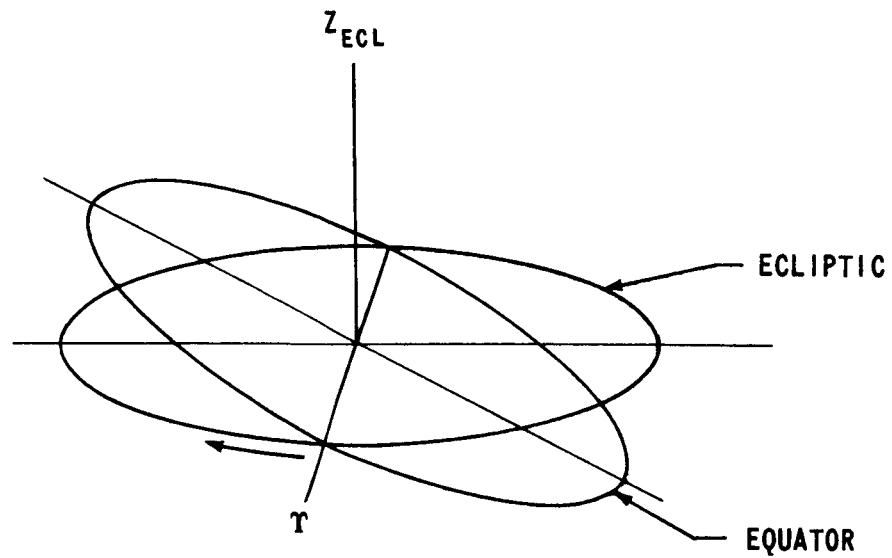
FIGURE 8

The gravitational pull of the planets on the earth causes the ecliptic plane to rotate toward the equatorial plane about a line in the ecliptic approximately 6° westward of the equinox (see Figure 9).
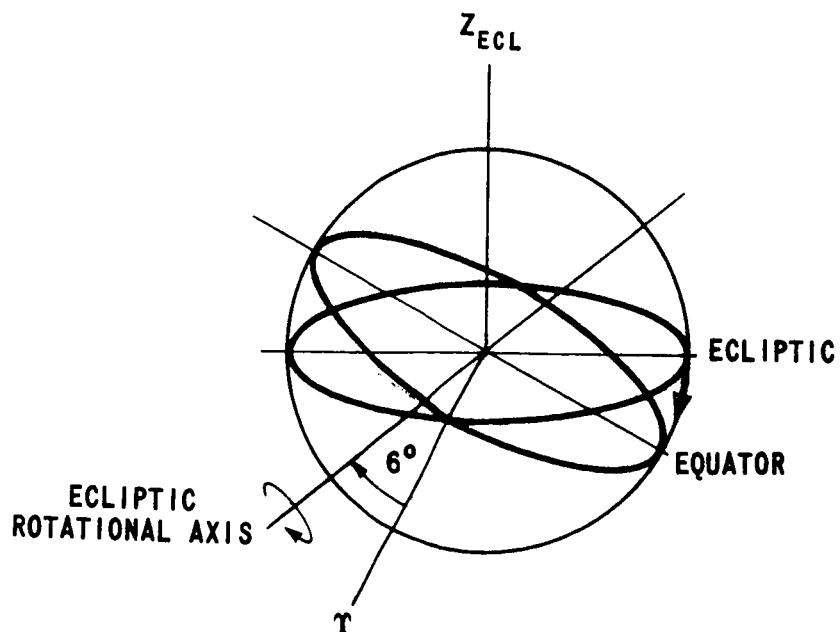

FIGURE 9

This rotation, called planetary precession, moves the mean equinox at a rate of 12" per century eastward along the ecliptic and decreases the obliquity at a rate of 47" per century. The combination of luni-solar precession and planetary precession is known as general precession.

The mean coordinates of one epoch can be related rigorously to those of another epoch by Euler angles of rotation $\zeta, z,$ and θ. These angles include the effects of both luni-solar and planetary precession. They are as follows:

$$\zeta = (2304\overset{''}{.}250 + 1\overset{''}{.}396\, T_o)\, T + 0\overset{''}{.}302T^2 + 0\overset{''}{.}018T^3$$

$$z = \zeta + 0\overset{''}{.}791T^2$$

$$\theta = (2004\overset{''}{.}682 - 0.853T_o)\, T - 0\overset{''}{.}426T^2 - 0\overset{''}{.}042T^3$$

where T_o is the initial epoch measured in tropical centuries since 1900.0 and T is the final epoch measured in tropical centuries since T_o. The .0 after the 1900 denotes the Besselian New Year, not the calendar New Year.

The transformation matrix relating X_o, Y_o, Z_o, the mean coordinates of one epoch, to X, Y, Z, the mean coordinates of another epoch, is as follows:

	X_o	Y_o	Z_o
X	$c\zeta\ c\theta\ cz - s\zeta\ sz$	$-s\zeta\ c\theta\ cz - c\zeta\ sz$	$-s\theta\ cz$
Y	$c\zeta\ c\theta\ sz +$	$-s\zeta\ c\theta\ sz + c\zeta c\ z$	$-s\theta\ cz$ (27)
Z	$c\zeta\ s\theta$	$-s\zeta\ s\theta$	$c\theta$

where s and c stand for sine and cosine respectively.

If the difference between epochs is a year or less, an approximation for the precession p can be used with negligible error. In the ecliptic coordinates, the annual westward shift in longitude is given by

$$p = 50\overset{''}{.}2564 - 0.0222T \tag{28}$$

where T is tropical centuries from 1900.0. The latitude remains unchanged.

The star's apparent place referred to the mean equator-equinox coordinate system of the nearest BNY can be computed by equation 22 of Section 2.4. If the event time <u>follows</u> the nearest BNY, one need only transform $\bar{R}$, the radius vector from the sun to the earth, from the ecliptic coordinate system into the mean equator-equinox coordinate system. The star's mean place and $\bar{R}$ are both referred to coordinate systems whose epochs are the nearest BNY. However, if the event time <u>precedes</u> the nearest BNY, $\bar{R}$, which is taken from the A.E. catalog of the current year, will be referred to the ecliptic coordinate system whose epoch precedes the nearest BNY by exactly one tropical year. It is, then, necessary to subtract equation 28, the precession in ecliptic coordinates, from the longitude of $\bar{R}$ given in the A.E. before $\bar{R}$ can be transformed into the mean equator-equinox coordinate system.

4.0 TIME

Two motions of the earth are used to measure time. The earth's rotation about its axis defines a day, and the earth's orbit around the sun defines a year.

To measure time, a scale of time measurement, a reference, and a time reckoner are required. The scale of time measure is the earth's equator which can be likened to the dial of a clock. The scale is arbitrarily divided into hours, minutes, and seconds such that 360° corresponds to 24 hours, one hour contains 60 minutes, and one minute contains 60 seconds (Figure 10). The scale starts at some reference. For local time measurement, the observer's meridian (M) is the reference, and for an absolute (independent of the observer's location) system of time measure, the Greenwich meridian is arbitrarily chosen to be the reference.
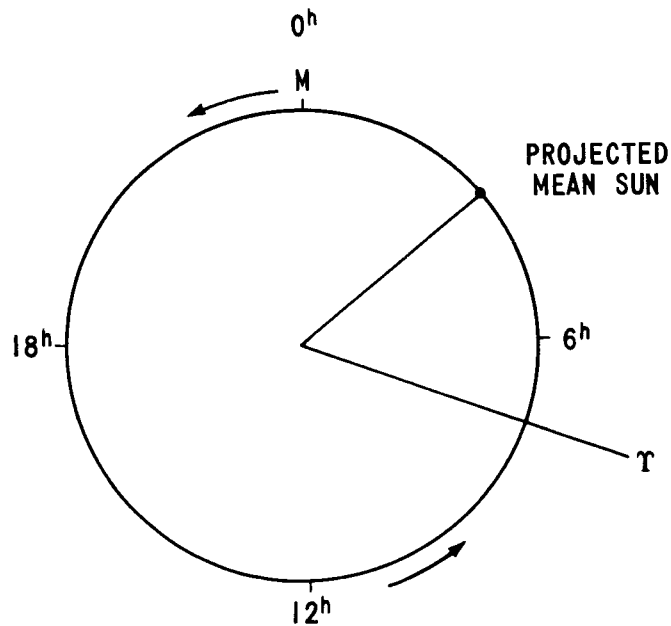
FIGURE 10

Time is indicated by the position of some fictitious point in the heavens projected on the equator. This point is the time reckoner, like the hands of a clock. Two time reckoners are used, the mean equinox and the mean sun defined later in Section 4.2. 24 hours of apparent motion of the mean equinox along the equator define a sidereal day, the fundamental unit of sidereal time (S.T.). 24 hours of apparent motion of the mean sun along the equator define a mean day, the fundamental unit of universal time (U.T.).

Both sidereal and universal time are affected by the earth's variable rate of rotation. Hence, an absolutely uniform measure of time called ephemeris time (E.T.), is defined to correspond to the parameter of time in all dynamical equations of motion. Therefore, the earth's orbit around the sun is the basis of ephemeris time measure. 360° of apparent motion of the mean sun relative to the mean equinox of date measured along the equator is used as the fundamental unit of ephemeris time. This unit of time is called the tropical year.

4.1 Sidereal Time

The local sidereal time (L.S.T.) is equal to the hour angle of the mean equinox as is shown in Figure 11.
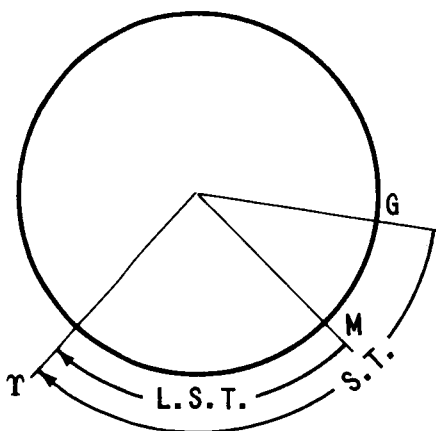
FIGURE II

where M is a meridian. The Greenwich sidereal time (or simply sidereal time) is the arc Gϓ.

Practical methods of time measurements based on observations of the sun and the stars are such that the hour angle of the true equinox rather than the mean equinox is determined. The hour angle of the true equinox is called the apparent (or true) sidereal time (A.S.T.). Because of nutation, apparent sidereal time is not a uniform measure of time and can differ from sidereal time by as much as $1^S.2$. The two times are related by the "equation of the equinoxes" formerly called the "nutation in right ascension" (α_{nut}), given below:

$$\alpha_{nut.} = A.S.T. - S.T. \tag{28}$$

As a matter of convenience, the "equation of equinoxes" is tabulated for both the long and short period terms of nutation in the APFS. Then, the apparent sidereal time minus the short period terms of the "equation of the equinoxes" corresponds to the right ascension of the star's apparent place given in the APFS. This has been done to facilitate the interpolation of the 10 day tables of star apparent places in the APFS.

The difference in length between an apparent sidereal day and a mean sidereal day is very small. This difference varies from $0^S.0005$ to $0^S.004$, and can be ignored in converting from apparent to mean sidereal times.

Sidereal time is a useful system of astronomical time measurement because the right ascension of a celestial body is numerically equal to the S.T. at a meridian when the body transits* the meridian. The more difficult problem of determining the time of transit in U.T. is considered in Appendix D, titled the "Time of Greenwich Transit", of this memorandum.

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*A celestial body is said to "transit" a meridian when the apparent position of the body lies on some extended straight line that originates from the center of the earth and passes through that meridian.

4.2 Universal Time

The mean sun rather than the true sun is the basis for universal time measure. Because the earth's orbit is eccentric, the velocity of the earth's orbit varies. Thus, the period of the apparent rotation (complete rotation relative to a meridian defines an apparent solar day) of the true sun varies from day to day. Even if the earth's orbit were circular, the duration of an apparent solar day would vary because the earth travels along the ecliptic rather than the equator. At the equinoxes, part of the earth's motion is north or south (see Figure 12).
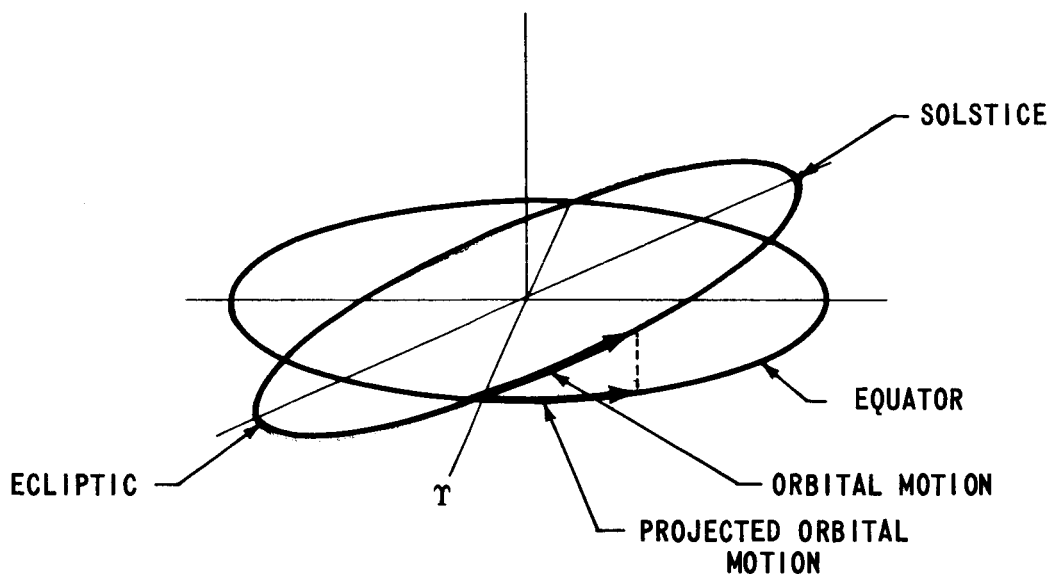
FIGURE 12

On the other hand, at the solstices (line on the ecliptic whose angle with the equator is greatest), the earth's motion is entirely eastward and since meridians are closer together at solstices than at the equator, the projected motion along the equator is increased (see Figure 13).
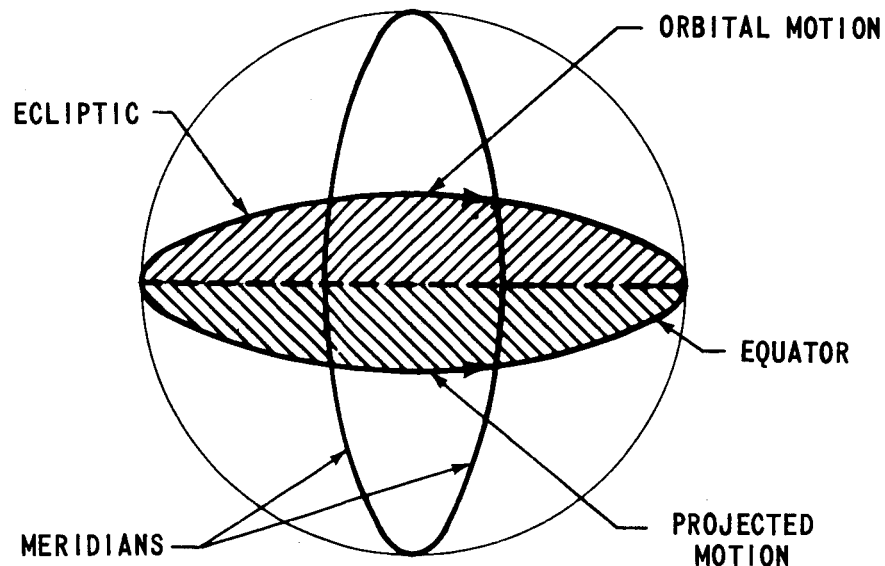
FIGURE 13

This effect makes the duration of an apparent solar day at the solstices longer than one near the equinoxes. Therefore, the true sun could not be used effectively as a measure of time. To make the length of days equal to each other, Newcomb defined a fictitious body called the mean sun whose right ascension (R.A.M.S.) referred to the mean equinox of date is given by

$$R.A.M.S. = 18^h\ 38^m\ 45^s.836 + 86401^s.8452T$$

$$+9^s.29 \times 10^{-6}\ T^2$$

(29)

where $18^h 38^m 45\overset{s}{.}836$ was the right ascension of the mean sun at the epoch Jan. $0\overset{d}{.}5$ 1900; T is the number of Julian years since that epoch. A Julian year is defined to contain exactly 365.25 mean days.

Because of the increasing right ascension of the mean sun, the mean day is longer than the sidereal day as is shown in Figure 14.
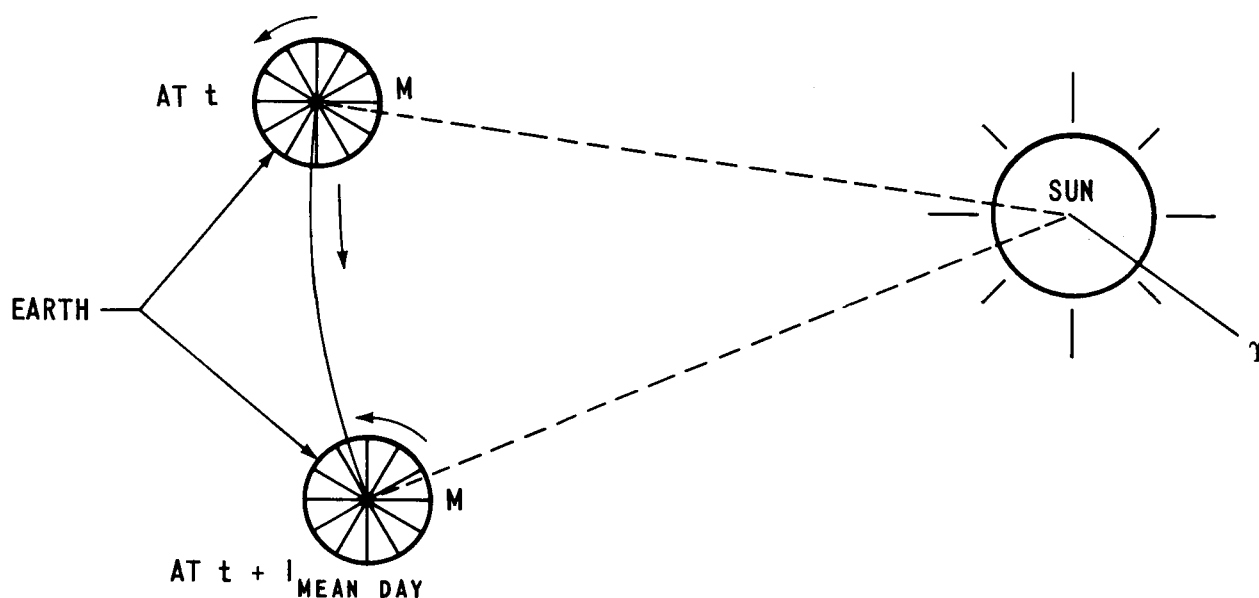
FIGURE 14

But the mean day and the sidereal day are rigorously related in S.T. by Newcomb's expression for the mean sun. The mean day is equal to one sidereal day plus the change in right ascension of the mean sun in a mean day.

$$1_{\text{mean day}} = 24^h + \frac{86401.84542}{365.25} = 86636\overset{s}{.}556 \text{ S.T.}$$

or

$$24^h \text{ U.T.} = 24^h 3^m 56\overset{s}{.}556 \text{ S.T.}$$

Universal time is equal to the hour angle of the mean sun (H.A.M.S.) measured from the Greenwich (G) meridian, plus 12^h (Figure 15).
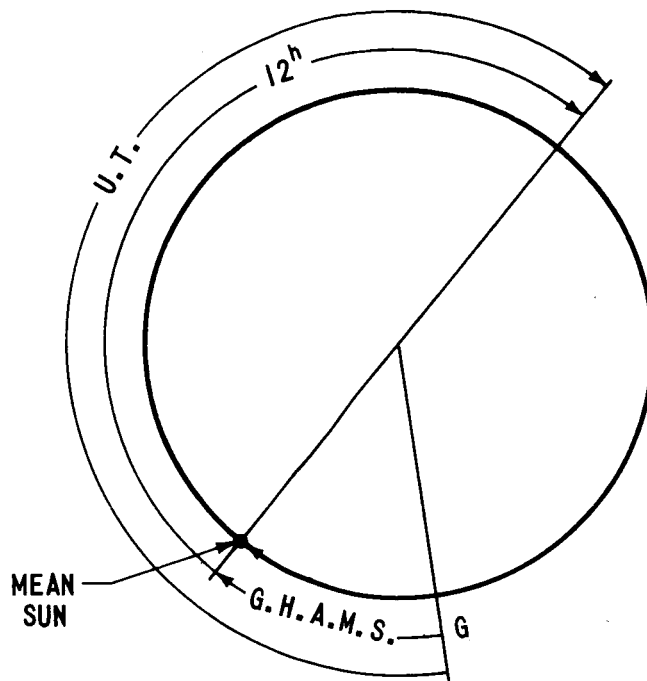
FIGURE 15

Hence,

$$U.T. = G.H.A.M.S. + 12^h \qquad\qquad (30)$$

The hour angle of the mean sun measured from any other meridian yields the local mean time at that meridian.

An entire system of astronomical time reckoning can be based on the mean day. The count of mean days (or Julian days) is chosen to begin at a time meant to predate all recorded history. That epoch is January $0^d.5$, 4713 B.C. The number of days since then determines the Julian Date. This system is advantageous for many astronomical computations.

4.3 Tropical Year

The tropical year is equal to the interval during which the mean sun's right ascension increases by 360°. Specifically, the tropical year at January 0, 1900 is equal to

$$\frac{360 \times 60 \times 60}{86401.84542 \times 15} \times 365.25 = 365.24219879 \text{ mean days}$$

The repeating cycle of the seasons corresponds directly to the tropical year.

The tropical year begins when the right ascension of the mean sun is exactly $18^h\ 40^m$. This is called the Besselian New Year (BNY) after the German astronomer, Bessel, who introduced this procedure into astronomical practice. This BNY is always within a day or two of the calendar New Year.

The calendar year is equal to 365.25 mean days. However, by a system of designating certain years as common years and others as leap years, the calendar is made to keep pace with the tropical year fairly accurately. Tables relate calendar date with the Julian systems, except that while a calendar day begins at midnight, the Julian day begins at noon.

4.4 Ephemeris Time

The unit of ephemeris time (E.T.) is the tropical year of January $0^d.5$, 1900 which contains

$$365.24219879 \times 86400 = 31556925.9747 \text{ ephemeris seconds}.$$

The ephemeris second, so defined, has been adopted as the fundamental invariable unit of time by the Comité International de Poids et Measures in 1957.

Ephemeris Time is obtained as a correction, ΔT, to universal time such that

$$E.T. = U.T. + \Delta T$$

ΔT is determined empirically, to make observations of the moon and the planets agree as closely as possible with orbital theory. Because E.T. corresponds to the parameter of time in orbital equations, the tables of the sun and the planets, given in the "American Ephemeris", are specified in E.T.

5.0   STAR PLACE COMPUTATION

The star's apparent place, as seen from geocenter through a hypothetical vacuum at some given time, referred to the mean geocentric equator-equinox coordinates of the nearest Besselian New Year (BNY), is the result obtained by the methods of this section.   One method starts with the star's apparent place given in the "Apparent Places of Fundamental Stars." The other one starts with the star's mean place given in either the APFS or the "American Ephemeris." A computer program has been written for each method and is available upon request. As an example, the apparent place of α Tauri is found at $8^d.7333$, May 1968 U.T. using both these programs.

The procedure using the apparent place of a star differs significantly from that using the mean place of a star. In the apparent place data method, one already has the apparent place of the star. Interpolation is required to obtain the star's apparent place ($\bar{u}_e$) at the desired time, and transformations are required to obtain the star's place in the desired coordinate system. On the other hand, in the mean place data method the star's place is already referred to the desired coordinate system. However, one must correct the star's mean place for certain physical phenomena, discussed in Section 2 of this memorandum, to obtain the star's apparent place ($\bar{u}_e$).

5.1   Reduction From Apparent Place Data

The "Apparent Places of Fundamental Stars" (APFS) gives the apparent place of the star as seen from geocenter through a fictitious, transparent, and non-atmospheric earth at dates (in ten day intervals of universal time) of upper transit at Greenwich referred to the true geocentric equator-equinox coordinate system of date, but with the omission of the short period terms of nutation. This is the starting point, and the following procedure is required.

5.1.1   Summary of Computation Procedure

1.   Find the time of Greenwich transit of the star for the entries nearest to the desired time.

2.   Interpolate the star's right ascension and declination to the desired time.

3.   Insert the short period terms of nutation.

4.   Find the X, Y, Z components of the star place.

5.   Apply nutation to go from the true equator-equinox coordinate system of date to the mean equator-equinox coordinate system of date.

6. Apply precession to go from the mean equator-equinox coordinate system of date to the mean equator-equinox coordinate system at the nearest BNY.

5.1.2 Input

1. α_0 = right ascension of the star in the APFS in hours, minutes, and seconds for the nearest catalog date preceding the event time.

2. δ_0 = declination of the star corresponding to α_0 in degrees, minutes, and seconds.

3. $A.S.T._0$ = apparent sidereal time measured from Greenwich at 0^h U.T. in hours, minutes, seconds of S.T., corresponding to α_0 and δ_0.

4. $\alpha_{nut.\ short\ 0}$ = short period "equation of the equinoxes" corresponding to $A.S.T._0$, in seconds of S.T.

5. d_0 = date in years, months, and days of U.T. corresponding to the $A.S.T._0$ and $\alpha_{nut.\ short\ 0}$ in the table. It is also the nearest integer day following that for α_0 and δ_0.

6. α_1 = right ascension of a star in hours, minutes, and seconds for nearest catalog date following the event.

7. δ_1 = declination corresponding to α_1 in degrees, minutes, and seconds.

8. $A.S.T._1$ = apparent sidereal time measured from Greenwich at 0^h U.T. in hours, minutes, and seconds of S.T., corresponding to α_1 and δ_1.

9. $\alpha_{nut.\ short\ 1}$ = short period "equation of the equinox" corresponding to $A.S.T._1$ in seconds of S.T.

10. d_1 = date in years, months, days of U.T. corresponding to $A.S.T._1$; it is also the nearest integer day following that of α_1 and δ_1.

11. d_e = date of the desired event in years, months, and days, and fractional parts thereof.

12. d_p = date specified to nearest integer day prior to d_e in years, months, and days of U.T. or E.T. depending on the source of the data.

13. d_{BNY} = date of the nearest BNY in years, months, and days and fractional parts thereof of U.T.

14. $d\psi_0$ and $d\psi_1$ = short period terms of nutation in longitude in arc seconds given at d_p and $d_p + 1^d$ in U.T. respectively.

15. $d\epsilon_0$ and $d\epsilon_1$ = short period terms of nutation in obliquity in degree seconds at d_p and $d_p + 1^d$ in U.T. respectively.

16. $\Delta'_{-\frac{1}{2}}$, $\Delta'_{\frac{1}{2}}$ and $\Delta'_{\frac{3}{2}}$ are first differences of both the right ascension and the declination. The differences are defined below

$$f_{-1}$$
$$\Delta'_{-\frac{1}{2}}$$
$$f_0$$
$$\Delta'_{\frac{1}{2}}$$
$$f_1$$
$$\Delta'_{\frac{3}{2}}$$
$$f_2$$

where f stands for any function.

17. $\Delta\psi_0$ and $\Delta\psi_1$ = nutation in longitude given in arc seconds at d_p and $d_p + 1^d$ E.T. respectively.

18. B_o and B_1 = the negative of the nutation in obliquity at d_p and $d_p + 1^d$ in E.T. respectively. B_o and B_1 are Besselian day numbers.

19. $d\alpha (\psi)$, and $d\alpha (\epsilon)$ = the differential corrections to the right ascension used to insert the short period terms of nutation (see Section 3.1). They are specified in hour seconds.

20. $d\delta (\psi)$, and $d\delta (\epsilon)$ = the differential corrections to the declination used to insert the short period terms of nutation (see Section 3.1, p. 5). They are specified in arc seconds.

5.1.3   Input Data for Sample Computation

The right ascension and the declination of α Tauri taken from the APFS at approximately May (V2.6) $2^d.6$, 1968 U.T., the nearest catalog date <u>prior</u> to the desired event date of May $8^d.7333$, 1968 U.T. (d_e), are as follows:

$$\alpha_o = 4^h \; 34^m \; 04^s.311$$

$$\delta_o = 16° \; 26' \; 53''.07$$

and approximately May (V12.6) $12^d.6$ 1968 U.T., the nearest catalog date <u>following</u> the desired event time, are as follows

$$\alpha_1 = 4^h \; 34^m \; 04^s.326$$

$$\delta_1 = 16° \; 26' \; 53''.26$$

as can be seen on page 73 of the APFS (Figure 12).

The first differences (obtainable from APFS, p. 73, Figure 12) required for the interpolation of the right ascension and the declination are as follows:

	α	δ
	$\Delta'_{-\frac{1}{2}} = -0^s.031$	$\Delta'_{-\frac{1}{2}} = 0''.07$
	$\Delta'_{\frac{1}{2}} = 0^s.015$	$\Delta'_{\frac{1}{2}} = 0''.19$
	$\Delta'_{\frac{3}{2}} = 0^s.060$	$\Delta'_{\frac{3}{2}} = 0''.31$

The differential star constants required to insert short period terms of nutation are tabulated for each star in the APFS at the bottom of the page. For α Tauri, they are

$$d\alpha \ (\psi) = 0\overset{s}{.}068 \quad , \quad d\delta \ (\psi) = 0\overset{"}{.}15$$

and

$$d\alpha \ (\epsilon) = -0\overset{s}{.}007 \quad , \quad d\delta \ (\epsilon) = 0\overset{"}{.}93$$

The short period terms of nutation obtained from Table I of the APFS on page 478 (Figure 13) at May 8, 1968 U.T. (d_p) are

$$d\psi_0 = 0\overset{"}{.}028$$

$$d\epsilon_0 = 0\overset{"}{.}094$$

and at May 9, 1968 U.T. $(d_p + 1^d)$ are

$$d\psi_1 = -0\overset{"}{.}075$$

$$d\epsilon_1 = 0\overset{"}{.}112$$

The apparent sidereal time corresponding to α_0 and δ_0 is obtained from Table II of the APFS on page 481 (Figure 14) at $d_0 = $ May 3, 1968 U.T.

$$A.S.T._0 = 14^h \ 43^m \ 49\overset{s}{.}021$$

and the short period "equation of the equinoxes" at d_0 is

$$\alpha_{nut. \ short \ o} = 0\overset{s}{.}001$$

The apparent sidereal time corresponding to α_1 and δ_1 is obtained from Table II of the APFS on page 481 (Figure 14) at $d_1 = $ May 13 1968 U.T.

$$A.S.T._1 = 15^h \ 23^m \ 14\overset{s}{.}571$$

and the short period equation of the equinoxes at d_1 is

$$\alpha_{nut\ short\ 1} = -0\overset{s}{.}014$$

The total nutation in longitude ($\Delta\psi$) and in obliquity ($\Delta\epsilon = -B$) taken from pages 22 (see Figure 15) and 264 (see Figure 16), respectively, of the "American Ephemeris" at May 8, 1968 E.T. (d_p) are

$$\Delta\psi_0 = -6\overset{''}{.}097$$

$$B_0 = -8\overset{''}{.}773$$

and at May 9, 1968 E.T. ($d_p + 1$) are

$$\Delta\psi_1 = -6\overset{''}{.}182$$

$$B_1 = -8\overset{''}{.}775$$

5.1.4 Sample Computation

1) Find the apparent sidereal time including the long period terms of nutations at d_0 (A.S.T.$_{10}$).

$$A.S.T._{10} = A.S.T._0 - \alpha_{nut.\ short\ o}$$

$$A.S.T._{10} = 14^h\ 43^m\ 49\overset{s}{.}020$$

and at d_1 (A.S.T.$_{11}$)

$$A.S.T._{11} = A.S.T._1 - \alpha_{nut\ short\ 1}$$

$$A.S.T._{11} = 15^h\ 23^m\ 14\overset{s}{.}585$$

2) Find the time of Greenwich transit for α Tauri at nearest catalog dates preceding and following the event time (d_e) in Julian days. The time of transit preceding d_e is given by

$$d_0' = 0.9972697\ (\alpha_0 - A.S.T._{10}) + d_0$$

and thus

$$d_o' = 2439979.078 \text{ J.D.}$$

The time of transit following d_e is given by

$$d_1' = 0.9972697 \, (\alpha_1 - \text{A.S.T.}_{11}) + d_1$$

and thus

$$d_1' = 2439989.051 \text{ J.D.}$$

3) Interpolate the star's right ascension and declination to the event time d_e by Bessel's interpolation formula

$$f_1 = f_o + n \, \Delta_{\frac{1}{2}}' + B'' \, (\Delta_{\frac{3}{2}}' - \Delta_{-\frac{1}{2}}')$$

where

$$n = \frac{d_e - d_o'}{d_1' - d_o'} \qquad , \qquad B'' = \frac{n(n-1)}{4}$$

$$f_o = \alpha_o \text{ or } \delta_o \qquad , \qquad f_1 = \alpha_1 \text{ or } \delta_1$$

one obtains

$$\alpha_1 = 4^h \, 34^m \, 04^s.313$$

and

$$\delta_1 = 16° \, 26' \, 53''.168$$

4) Compute the differential correction to the right ascension and to the declinations using the short period terms of nutation at the nearest integer day preceding ($\Delta\alpha_0$ and $\Delta\delta_0$) the event time d_e and the nearest integer day following ($\Delta\alpha_1$ and $\Delta\delta_1$) the event by

$$\Delta\alpha = d\alpha\ (\psi)d\ \psi + d\alpha(\epsilon)d\epsilon$$

$$\Delta\delta = d\delta\ (\psi)d\ \psi + d\delta(\epsilon)d\epsilon$$

Interpolate the differential corrections linearly to the event time and add as shown

$$\alpha = \alpha_i + \Delta\alpha_0 + (d_e - d_p)\ (\Delta\alpha_1 - \Delta\alpha_0)$$

$$\alpha = 4^h\ 34^m\ 04\overset{s}{.}310$$

$$\delta = \delta_i + \Delta\delta_0 + (d_e - d_p)\ (\Delta\delta_1 - \Delta\delta_0)$$

$$\delta = 16°\ 26'\ 53\overset{''}{.}260$$

5) Compute the components of the interpolated star place in true equatorial coordinates at d_e

$$X = \cos\delta\cos\alpha = 0.35123137$$

$$Y = \cos\delta\sin\alpha = 0.89244908$$

$$Z = \sin\delta\qquad = 0.28314508$$

6) Compute the mean obliquity by using a formula given on page 98 of the "Explanatory Supplement to the Ephemeris"

$$\epsilon_0 = 23\overset{°}{.}452294 - 0\overset{°}{.}0035626D +$$

$$- 0\overset{°}{.}000000123D^2 + 0\overset{°}{.}0000000103D^3$$

where

$$D = 10^{-4}\ (d_e - d_{Jan\ 0.5,\ 1900})$$

d_e and $d_{Jan\ 0.5,\ 1900}$ are converted into Julian Dates

$$\text{J.D. of } d_{Jan\ 0.5\ 1900} = 2415020.0$$

$$\text{J.D. of } d_e = 2439985.233$$

The mean obliquity is

$$\epsilon_o = 23\overset{o}{.}443410$$

7) Compute the nutation in longitude and in obliquity at the event time. Since the tables are in ephemeris time (E.T.), one determines the event time in E.T. by

$$d_e' = d_e + \Delta T$$

where for 1968, $\Delta T = 0\overset{d}{.}0004$.

The nutation in longitude is given by

$$\Delta\psi = \Delta\psi_0 + (d_e' - d_p)(\Delta\psi_1 - \Delta\psi_0)$$

and thus

$$\Delta\psi = 6\overset{"}{.}182$$

The negative of the nutation in obliquity is given by

$$B = B_0 + (d_e' - d_p)(B_1 - B_0)$$

which yields

$$B = -8\overset{"}{.}775$$

and

$$\Delta\epsilon = 8\overset{"}{.}775$$

8) Apply the nutation matrix to convert the star place from the true equator-equinox coordinates $(\bar{R})$ of date d_e to mean $(\bar{R}_m)$, of date

$$\bar{R}_m = [N]\,\bar{R}$$

where

$$[N] = \begin{bmatrix} 1.0 & -0.27396253 \times 10^{-4} & 0.11880068 \times 10^{-4} \\ 0.27396253 \times 10^{-4} & 1.0 & 0.42539494 \times 10^{-4} \\ 0.11880068 \times 10^{-4} & -0.42539494 \times 10^{-4} & 1.0 \end{bmatrix}$$

Then

$$X_m = 0.35120356$$

$$Y_m = 0.89247074$$

$$Z_m = 0.28311129$$

9) Compute the Euler angles required for the precession matrix using Newcomb's formula (section 3.2), here converted to degree and days since Jan 0^d.0, 1950 (J.D. 2433281.5)

$$\zeta = (0°.1752983 + 29°.0694 \times 10^{-6}D_0)D$$
$$+ 6°.289 \times 10^{-6}D^2 + 0°.1025 \times 10^{-6}D^3$$

$$z = \zeta + 16°.4694 \times 10^{-6}D^2$$
$$\theta = (0°.1524946 - 17°.761 \times 10^{-6}D_0)D$$
$$- 8°.869 \times 10^{-6}D^2 - 0°.2394 \times 10^{-6}D^3$$

where $D_0 = 10^{-4}d_0$ and $D = 10^{-4}d$. d_0 and d are defined by

$$d_0 = d_e - 2433281.5$$

$$d = d_{BNY} - d_0$$

One obtains for the Euler angles

$$\zeta = -3.9304111 \times 10^{-5} \text{ radians}$$

$$z = -3.9304159 \times 10^{-5} \text{ radians}$$

$$\theta = -3.4184843 \times 10^{-5} \text{ radians}$$

10) Apply the precession matrix [P] to transform from the mean equator-equinox coordinates ($\bar{R}_m$) of date into the mean equator-equinox coordinates of the nearest BNY ($\bar{R}_{BNY}$) by

$$\bar{R}_{BNY} = [P]\bar{R}_m$$

where

$$P = \begin{bmatrix} 1.0 & -0.78608270 \times 10^{-4} & -0.34184843 \times 10^{-4} \\ 0.78608270 \times 10^{-4} & 1.0 & -0.13436065 \times 10^{-8} \\ 0.34184843 \times 10^{-4} & -0.13436049 \times 10^{-8} & 1.0 \end{bmatrix}$$

One obtains for the final answer

$$X = 0.35128339$$

$$Y = 0.89244314$$

$$Z = 0.28309928$$

APPARENT PLACES OF STARS, 1968

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AT UPPER TRANSIT AT GREENWICH

No.	168		170		169		172	
Name	a Teuri (Aldebaran)		v^2 Eridani		r Eridani		53 Eridani*	
Mag. Spect.	1.06	K5	3.88	K0	4.12	B2	3.98	K0
U. T.	R. A.	Dec.	R. A.	Dec.	R. A.	Dec.	R. A.	Dec.
	$4^h 34^m$	$+16° 26'$	$4^h 34^m$	$-30° 37'$	$4^h 34^m$	$-3° 24'$	$4^h 36^m$	$-14° 21'$
I -6.1	05.600 + 62	55.06 - 19	19.514 + 11	35.30 - 234	43.866 + 49	54.58 - 132	43.690 + 30	51.84 - 176
I 3.9	05.619 - 23	54.87 - 19	19.479 - 79	37.66 - 206	43.874 - 32	55.82 - 111	43.687 - 44	53.60 - 157
I 13.9	05.596 - 63	54.68 - 19	19.400 - 118	39.72 - 174	43.842 - 69	56.93 - 96	43.643 - 62	55.17 - 133
I 23.9	05.533 - 100	54.49 - 19	19.282 - 156	41.46 - 137	43.773 - 104	57.89 - 79	43.561 - 116	56.50 - 108
II 2.8	05.433 - 131	54.30 - 19	19.126 - 185	42.83 - 93	43.669 - 133	58.68 - 59	43.445 - 145	57.58 - 78
II 12.8	05.302 - 151	54.11 - 21	18.941 - 203	43.76 - 53	43.536 - 151	59.27 - 41	43.300 - 144	58.36 - 49
II 22.8	05.151 - 165	53.90 - 21	18.738 - 217	44.29 - 10	43.385 - 165	59.68 - 20	43.136 - 170	58.85 - 18
III 3.7	04.986 - 166	53.69 - 20	18.521 - 217	44.39 + 36	43.220 - 166	59.88 + 2	42.958 - 178	59.03 + 13
III 13.7	04.820 - 157	53.49 - 19	18.304 - 207	44.03 + 75	43.054 - 157	59.86 + 21	42.780 - 170	58.90 + 43
III 23.7	04.663 - 139	53.30 - 16	18.097 - 188	43.28 + 117	42.897 - 141	59.65 + 43	42.610 - 154	58.47 + 73
IV 2.7	04.524 - 108	53.14 - 11	17.909 - 159	42.11 + 156	42.756 - 113	59.22 + 66	42.456 - 126	57.74 + 104
IV 12.6	04.416 - 74	53.03 - 3	17.750 - 123	40.55 + 189	42.643 - 81	58.56 + 85	42.330 - 93	56.70 + 129
IV 22.6	04.342 - 31	53.00 + 7	17.627 - 82	38.66 + 221	42.562 - 41	57.71 + 106	42.237 - 54	55.41 + 156
{IV 2.6	04.311 + 15	53.07 + 19	17.545 - 33	36.45 + 248	42.521 + 3	56.65 + 126	42.183 - 10	53.85 + 177
{IV 12.6	04.326 + 60	53.26 + 31	17.512 + 13	33.97 + 268	42.524 + 45	55.39 + 144	42.173 + 33	52.06 + 197
IV 22.5	04.386 + 106	53.57 + 45	17.525 + 63	31.29 + 286	42.569 + 91	53.95 + 159	42.206 + 80	50.09 + 215
V 1.5	04.494 + 153	54.02 + 59	17.588 + 113	28.43 + 294	42.660 + 136	52.36 + 171	42.286 + 123	47.94 + 225
V 11.5	04.647 + 192	54.61 + 71	17.701 + 155	25.49 + 295	42.794 + 171	50.65 + 179	42.409 + 162	45.69 + 231
V 21.4	04.839 + 229	55.32 + 83	17.856 + 199	22.54 + 292	42.965 + 208	48.86 + 184	42.571 + 200	43.38 + 232
V 1.4	05.068 + 259	56.15 + 91	18.055 + 204	19.62 + 276	43.173 + 237	47.02 + 189	42.771 + 231	41.06 + 225
VI 11.4	05.327 + 282	57.06 + 97	18.289 + 244	16.86 + 256	43.410 + 260	45.19 + 176	43.002 + 256	38.81 + 212
VI 21.4	05.609 + 301	58.03 + 100	18.553 + 270	14.30 + 228	43.670 + 281	43.43 + 164	43.258 + 278	36.69 + 194
VI 31.3	05.910 + 313	59.03 + 99	18.843 + 306	12.02 + 190	43.951 + 291	41.79 + 147	43.536 + 291	34.75 + 168
VII 10.3	06.223 + 318	60.02 + 95	19.149 + 317	10.12 + 151	44.242 + 299	40.32 + 126	43.827 + 298	33.07 + 138
VII 20.3	06.541 + 321	60.97 + 88	19.466 + 324	08.61 + 102	44.541 + 302	39.06 + 100	44.125 + 304	31.69 + 102
VII 30.3	06.862 + 317	61.85 + 77	19.790 + 320	07.59 + 51	44.843 + 298	38.06 + 69	44.429 + 299	30.67 + 62
VIII 9.2	07.179 + 309	62.62 + 65	20.110 + 314	07.08 + 0	45.141 + 291	37.37 + 39	44.728 + 294	30.05 + 23
VIII 19.2	07.488 + 300	63.27 + 51	20.424 + 301	07.08 - 55	45.432 + 282	36.98 + 5	45.022 + 283	29.82 - 20
VIII 29.2	07.788 + 283	63.78 + 36	20.725 + 281	07.63 - 105	45.714 + 265	36.93 - 27	45.305 + 266	30.02 - 61
IX 9.1	08.071 + 269	64.14 + 24	21.006 + 261	08.68 - 151	45.979 + 250	37.20 - 56	45.571 + 250	30.63 - 98
IX 19.1	08.339 + 247	64.38 + 10	21.267 + 232	10.19 - 194	46.229 + 228	37.76 - 89	45.821 + 227	31.61 - 132
IX 29.1	08.586 + 221	64.48 + 0	21.499 + 209	12.13 - 226	46.457 + 204	38.59 - 106	46.048 + 200	32.93 - 149
X 8.1	08.807 + 196	64.48 - 9	21.699 + 166	14.39 - 251	46.661 + 177	39.65 - 122	46.248 + 172	34.53 - 180
X 18.0	09.003 + 162	64.39 - 15	21.865 + 125	16.90 - 267	46.838 + 145	40.87 - 134	46.420 + 138	36.33 - 194
X 28.0	09.165 + 127	64.24 - 19	21.990 + 83	19.57 - 269	46.983 + 110	42.21 - 138	46.558 + 102	38.27 - 198
XI 7.9	09.292 + 89	64.05 - 20	22.073 + 40	22.26 - 265	47.093 + 75	43.59 - 137	46.660 + 65	40.25 - 196
XI 17.9	09.381 + 46	63.85 - 22	22.113 - 7	24.91 - 251	47.168 + 30	44.96 - 130	46.725 + 22	42.21 - 183
XI 27.9	09.427 + 3	63.63 - 21	22.106 - 51	27.42 - 226	47.201 - 6	46.29 - 121	46.747 - 19	44.09 - 170
XI 37.9	09.432 - 37	63.42 - 21	22.055 - 94	29.68 - 198	47.195 - 45	47.50 - 109	46.729 - 57	45.79 - 152
Mean Place	04.892	46.97	18.314	36.92	43.070	59.85	42.777	55.66
sec δ, tan δ	+1.043	+0.295	+1.162	-0.592	+1.002	-0.060	+1.032	-0.256
dα(ψ),dδ(ψ)	+0.068	+0.15	+0.047	+0.15	+0.060	+0.14	+0.055	+0.14
dα(ε),dδ(ε)	-0.007	+0.93	+0.014	+0.93	+0.001	+0.93	+0.006	+0.93
Bsis. Trans.	November 29		November 29		November 29		November 30	

FIGURE 12 -"APPARENT PLACES OF FUNDAMENTAL STARS"

470 　　　　 # TABLE I, 1968

SHORT-PERIOD TERMS OF NUTATION

Date	dψ	dε	Date	dψ	dε	Date	dψ	dε	Date	dψ	dε
	(0.001)			(0.001)			(0.001)			(0.001)	
Jan. 0	+85	−110	Feb. 15	+139	+65	Apr. 1	−223	+7	May 17	+280	−69
1	+209	−88	16	+58	+100	2	−225	−35	18	+332	−21
2	+289	−46	17	−47	+111	3	−191	−71	19	+325	+29
3	+312	+4	18	−149	+96	4	−129	−95	20	+268	+70
4	+280	+50	19	−219	+57	5	−50	−104	21	+176	+96
5	+205	+84	20	−239	+6	6	+32	−93	22	+69	+103
6	+104	+101	21	−199	−47	7	+101	−65	23	−36	+92
7	−3	+99	22	−110	−88	8	+142	−22	24	−124	+66
8	−100	+80	23	+9	−108	9	+141	+28	25	−182	+28
9	−173	+48	24	+132	−103	10	+94	+74	26	−203	−13
10	−213	+7	25	+231	−75	11	+7	+106	27	−188	−53
11	−214	−35	26	+288	−31	12	−101	+113	28	−141	−83
12	−178	−71	27	+293	+18	13	−198	+91	29	−72	−100
13	−112	−95	28	+247	+62	14	−253	+44	30	+6	−99
14	−26	−103	29	+163	+92	15	−244	−15	31	+77	−81
15	+63	−92	Mar. 1	+58	+104	16	−169	−68	June 1	+127	−47
16	+135	−62	2	−49	+96	17	−46	−103	2	+143	−4
17	+174	−18	3	−140	+72	18	+93	−111	3	+118	+42
18	+167	+32	4	−202	+35	19	+216	−92	4	+54	+82
19	+113	+75	5	−229	−7	20	+297	−54	5	−42	+106
20	+22	+103	6	−218	−48	21	+326	−6	6	−152	+106
21	−85	+107	7	−173	−81	22	+302	+41	7	−245	+81
22	−182	+86	8	−101	−100	23	+233	+78	8	−294	+33
23	−243	+45	9	−16	−102	24	+137	+100	9	−277	−26
24	−251	−8	10	+69	−85	25	+28	+102	10	−191	−78
25	−199	−59	11	+134	−50	26	−75	+88	11	−55	−110
26	−97	−96	12	+164	−3	27	−156	+58	12	+99	−113
27	+32	−110	13	+147	+48	28	−205	+18	13	+230	−87
28	+157	−98	14	+82	+91	29	−218	−24	14	+312	−40
29	+253	−63	15	−17	+112	30	−194	−62	15	+331	+13
30	+297	−15	16	−124	+106	May 1	−140	−89	16	+292	+60
31	+286	+34	17	−207	+74	2	−67	−102	17	+211	+91
Feb. 1	+226	+74	18	−240	+22	3	+12	−97	18	+106	+104
2	+133	+98	19	−211	−34	4	+81	−74	19	−1	+97
3	+25	+103	20	−126	−80	5	+126	−38	20	−94	+74
4	−77	+89	21	−6	−106	6	+136	+9	21	−159	+38
5	−159	+60	22	+121	−106	7	+103	+57	22	−190	−3
6	−210	+21	23	+227	−83	8	+28	+94	23	−183	−44
7	−224	−21	24	+292	−43	9	−75	+112	24	−142	−77
8	−200	−60	25	+308	+6	10	−181	+102	25	−77	−97
9	−143	−89	26	+273	+51	11	−258	+65	26	+2	−100
10	−63	−103	27	+197	+85	12	−278	+10	27	+77	−86
11	+28	−98	28	+96	+102	13	−228	−49	28	+133	−55
12	+110	−74	29	−14	+101	14	−116	−95	29	+158	−13
13	+164	−33	30	−111	+81	15	+30	−114	30	+142	+32
14	+176	+17	31	−185	+48	16	+173	−104	July 1	+84	+73
15	+139	+65	Apr. 1	−223	+7	17	+280	−69	2	−6	+100

Corrections to apparent places of 10-day stars are given by:

$$\Delta\alpha = d\alpha(\psi) \cdot d\psi + d\alpha(\varepsilon) \cdot d\varepsilon \qquad \Delta\delta = d\delta(\psi) \cdot d\psi + d\delta(\varepsilon) \cdot d\varepsilon$$

where dψ and dε are to be taken from the table above, and their coefficients are tabulated under each star.

FIGURE 13 − "APPARENT PLACES OF FUNDAMENTAL STARS"

TABLE II, 1968
SIDEREAL TIME AT 0ʰ U.T.

Date	Sidereal Time Apparent	Mean	Equation of Equinoxes Long-Period	Short-Period	Date	Sidereal Time Apparent	Mean	Equation of Equinoxes Long-Period	Short-Period
			(0.001)					(0.001)	
Apr. 1	12ʰ 37ᵐ 39.254	39.628	−360	−14	May 17	15ʰ 39ᵐ 00.830	01.175	−362	+17
2	12 41 35.808	36.183	−362	−14	18	15 42 57.391	57.730	−360	+20
3	12 45 32.364	32.739	−363	−12	19	15 46 53.948	54.285	−358	+20
4	12 49 28.922	29 294	−365	− 8	20	15 50 50.501	50.841	−356	+16
5	12 53 25.480	25.849	−366	− 3	21	15 54 47.053	47.396	−354	+11
6	12 57 22.039	22.405	−367	+ 2	22	15 58 43.604	43.952	−352	+ 4
7	13 01 18.598	18.960	−369	+ 6	23	16 02 40.155	40.507	−349	− 2
8	13 05 15.154	15.516	−370	+ 9	24	16 06 36.708	37.062	−347	− 8
9	13 09 11.708	12.071	−371	+ 9	25	16 10 33.262	33.618	−345	−11
10	13 13 08.260	08.626	−372	+ 6	26	16 14 29.818	30.173	−342	−12
11	13 17 04.809	05.182	−373	0	27	16 18 26.377	26.728	−340	−11
12	13 21 01.356	01.737	−374	− 6	28	16 22 22.938	23.284	−337	− 9
13	13 24 57.905	58.292	−375	−12	29	16 26 19.500	19.839	−335	− 4
14	13 28 54.456	54.848	−376	−15	30	16 30 16.063	16.394	−332	0
15	13 32 51.011	51.403	−377	−15	31	16 34 12.625	12.950	−329	+ 5
16	13 36 47.570	47.958	−378	−10	June 1	16 38 09.187	09.505	−326	+ 8
17	13 40 44.132	44.514	−379	− 3	2	16 42 05.746	06.061	−324	+ 9
18	13 44 40.696	41.069	−379	+ 6	3	16 46 02.302	02.616	−321	+ 7
19	13 48 37.258	37.625	−380	+13	4	16 49 58.857	59.171	−318	+ 3
20	13 52 33.818	34.180	−380	+18	5	16 53 55.409	55.727	−315	− 3
21	13 56 30.375	30.735	−381	+20	6	16 57 51.961	52.282	−312	− 9
22	14 00 26.928	27.291	−381	+19	7	17 01 48.514	48.837	−309	−15
23	14 04 23.479	23.846	−381	+14	8	17 05 45.069	45.393	−306	−18
24	14 08 20.028	20.401	−381	+ 8	9	17 09 41.629	41.948	−302	−17
25	14 12 16.577	16.957	−381	+ 2	10	17 13 38.192	38.503	−299	−12
26	14 16 13.126	13.512	−381	− 5	11	17 17 34.759	35.059	−296	− 3
27	14 20 09.677	10.067	−381	−10	12	17 21 31.327	31.614	−293	+ 6
28	14 24 06.229	06.623	−381	−13	13	17 25 27.894	28.170	−290	+14
29	14 28 02.784	03.178	−381	−13	14	17 29 24.458	24.725	−286	+19
30	14 31 59.341	59.734	−380	−12	15	17 33 21.017	21.280	−283	+20
May 1	14 35 55.900	56.289	−380	− 9	16	17 37 17.574	17.836	−280	+18
2	14 39 52.461	52.844	−380	− 4	17	17 41 14.127	14.391	−277	+13
3	14 43 49.021	49.400	−379	+ 1	18	17 45 10.680	10.946	−273	+ 6
4	14 47 45.582	45.955	−378	+ 5	19	17 49 07.232	07.502	−270	0
5	14 51 42.141	42.510	−378	+ 8	20	17 53 03.785	04.057	−267	− 6
6	14 55 38.697	39.066	−377	+ 8	21	17 57 00.339	00.612	−263	−10
7	14 59 35.252	35.621	−376	+ 6	22	18 00 56.896	57.168	−260	−12
8	15 03 31.804	32.176	−375	+ 2	23	18 04 53.455	53.723	−257	−11
9	15 07 28.354	28.732	−374	− 4	24	18 08 50.017	50.279	−253	− 9
10	15 11 24.904	25.287	−372	−11	25	18 12 46.579	46.834	−250	− 5
11	15 15 21.456	21.843	−371	−16	26	18 16 43.143	43.389	−247	0
12	15 19 18.011	18.398	−370	−17	27	18 20 39.706	39.945	−243	+ 5
13	15 23 14.571	14.953	−368	−14	28	18 24 36.268	36.500	−240	+ 8
14	15 27 11.135	11.509	−367	− 7	29	18 28 32.828	33.055	−237	+10
15	15 31 07.701	08.064	−365	+ 2	30	18 32 29.386	29.611	−234	+ 9
16	15 35 04.267	04.619	−363	+11	July 1	18 36 25.941	26.166	−230	+ 5
17	15 39 00.830	01.175	−362	+17	2	18 40 22.494	22.721	−227	0

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FIGURE 14 —"APPARENT PLACES OF FUNDAMENTAL STARS"

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SUN, 1968

ΔΨ ↓

FOR 0ʰ EPHEMERIS TIME

Date	Longitude Mean Equinox of 1968.0	Redn. to App. Long.	Latitude Ecliptic of 1968.0	Latitude Ecliptic of 1950.0	Date	Hor. Par.	Prec. in Long.	Nutation in Long.	Obl. of Ecliptic 23°26′
Apr. 1	11 20 00.6 3551.8	–14.1	+0.01	2.43	+0.04	8.80	+12.486	6.112	45.428
2	12 19 12.4 3549.5	14.0	.11	2.46	.15	8.80	12.624	6.139	45.380
3	13 18 21.0 3547.3	13.8	.20	2.51	.24	8.80	12.761	6.129	45.337
4	14 17 29.2 3545.0	13.7	.27	2.58	.31	8.80	12.899	6.091	45.305
5	15 16 34.2 3542.6	13.5	.31	2.68	.35	8.79	13.037	6.035	45.289
6	16 15 36.8 3540.4	–13.3	+0.33	2.80	+0.37	8.79	+13.174	–5.975	45.291
7	17 14 37.2 3538.1	13.1	.31	2.95	.36	8.79	13.312	5.927	45.311
8	18 13 35.3 3535.8	12.9	.26	3.13	.31	8.79	13.450	5.907	45.343
9	19 12 31.1 3533.5	12.8	.19	3.34	.24	8.79	13.587	5.927	45.384
10	20 11 24.6 3531.2	12.7	+.09	3.57	.14	8.78	13.725	5.993	45.410
11	21 10 15.8 3529.1	–12.7	0.03	3.82	+0.03	8.78	+13.863	–6.098	45.440
12	22 09 04.9 3527.0	12.6	.17	4.09	.11	8.78	14.000	6.223	45.435
13	23 07 51.9 3525.0	12.6	.31	4.36	.25	8.78	14.138	6.335	45.402
14	24 06 36.9 3523.1	12.5	.45	4.62	.39	8.77	14.276	6.405	45.343
15	25 05 20.0 3521.3	12.4	.59	4.88	.52	8.77	14.413	6.410	45.270
16	26 04 01.3 3519.5	–12.2	–0.70	–5.12	–0.63	8.77	+14.551	–6.347	45.204
17	27 02 40.8 3517.8	11.9	.79	5.34	.72	8.77	14.688	6.235	45.156
18	28 01 18.6 3516.2	11.7	.85	5.51	.77	8.76	14.826	6.106	45.133
19	28 59 54.8 3514.5	11.4	.88	5.66	.80	8.76	14.964	5.992	45.138
20	29 58 29.3 3512.9	11.2	.87	5.77	.79	8.76	15.101	5.919	45.162
21	30 57 02.2 3511.3	–11.0	–0.83	–5.85	–0.75	8.76	+15.239	–5.897	45.195
22	31 55 33.5 3509.6	10.9	.76	5.90	.67	8.75	15.377	5.926	45.226
23	32 54 03.1 3508.0	10.8	.67	5.91	.58	8.75	15.514	5.999	45.248
24	33 52 31.1 3506.2	10.8	.56	5.92	.46	8.75	15.652	6.097	45.254
25	34 50 57.3 3504.6	10.8	.43	5.90	.33	8.75	15.790	6.207	45.240
26	35 49 21.9 3502.8	–10.7	–0.30	–5.88	–0.20	8.74	+15.927	–6.310	45.210
27	36 47 44.7 3501.0	10.7	.17	5.86	.07	8.74	16.065	6.390	45.164
28	37 46 05.7 3499.1	10.6	–.05	5.84	+.06	8.74	16.202	6.436	45.107
29	38 44 24.8 3497.3	10.4	+.07	5.83	.17	8.74	16.340	6.444	45.048
30	39 42 42.1 3495.5	10.3	.16	5.84	.27	8.73	16.478	6.415	44.993
May 1	40 40 57.6 3493.5	–10.1	+0.24	–5.87	+0.35	8.73	+16.615	–6.354	44.950
2	41 39 11.1 3491.5	9.8	.29	5.91	.40	8.73	16.753	6.272	44.919
3	42 37 22.6 3489.7	9.6	.31	5.98	.43	8.73	16.891	6.183	44.907
4	43 35 32.3 3487.6	9.4	.31	6.08	.43	8.73	17.028	6.103	44.914
5	44 33 39.9 3485.8	9.2	.28	6.21	.40	8.72	17.166	6.045	44.933
6	45 31 45.7 3483.7	–9.0	+0.21	–6.36	+0.34	8.72	+17.304	–6.021	44.962
7	46 29 49.4 3481.8	8.9	.13	6.54	.25	8.72	17.441	6.039	44.993
→8	47 27 51.2 3479.9	8.8	+.01	6.74	.14	8.72	17.579	_6.097_	45.013
9	48 25 51.1 3478.1	8.7	–.12	6.95	+.02	8.72	17.717	_6.182_	45.013
10	49 23 49.2 3476.3	8.7	.25	7.17	–.12	8.71	17.854	6.268	44.987
11	50 21 45.5 3474.6	–8.6	–0.39	–7.39	0.25	8.71	+17.992	–6.324	44.933
12	51 19 40.1 3473.0	8.5	.52	7.60	.38	8.71	18.129	6.322	44.862
13	52 17 33.1 3471.6	8.2	.64	7.80	.49	8.71	18.267	6.248	44.785
14	53 15 24.7 3470.1	8.0	.73	7.96	.58	8.71	18.405	6.111	44.723
15	54 13 14.8 3468.9	7.6	.80	8.10	.65	8.70	18.542	5.938	44.688
16	55 11 03.7 3467.7	–7.3	–0.83	–8.21	–0.68	8.70	+18.680	–5.768	44.682
17	56 08 51.4	–7.1	–0.83	–8.28	–0.68	8.70	+18.818	–5.631	44.701

To obtain the longitude referred to the mean equinox of 1950.0, subtract 15′ 04″.9.

FIGURE 15 –"AMERICAN EPHEMERIS"

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BESSELIAN DAY NUMBERS, 1968
$-\Delta\epsilon$ FOR 0h EPHEMERIS TIME

Date	A	B	C	D	E	dψ	dε	τ	S.T.
					(0s·0001)	(0"·001)			h
Apr. 1	− 2·546	−9·141	−18·437	− 4·037	9	−223	+ 7	+0·2484	12·6
2	2·590	9·094	18·370	4·384	0	−225	− 35	·2511	12·7
3	2·649	9·052	18·297	4·728	0	−191	− 71	·2539	12·8
4	2·719	9·022	18·218	5·071	0	−129	− 95	·2566	12·8
5	2·796	9·007	18·134	5·411	0	− 50	− 104	·2593	12·9
6	+ 2·875	−9·010	−18·045	− 5·750	9	+ 32	− 93	+0·2621	13·0
7	2·949	9·031	17·950	6·086	9	+101	− 65	·2648	13·0
8	3·012	9·065	17·850	6·420	9	+142	− 22	·2675	13·1
9	3·059	9·107	17·746	6·752	9	+141	+ 28	·2703	13·2
10	3·087	9·143	17·636	7·080	9	+ 94	+ 74	·2730	13·2
11	− 3·100	−9·166	−17·522	− 7·407	− 9	+ 7	+106	+0·2758	13·3
12	3·105	9·162	17·404	7·731	9	−101	+113	·2785	13·4
13	3·116	9·130	17·280	8·052	9	−198	+ 91	·2812	13·4
14	3·143	9·072	17·152	8·371	9	−253	+ 44	·2840	13·5
15	3·196	9·001	17·020	8·688	9	−244	− 15	·2867	13·5
16	+ 3·276	−8·936	−16·883	− 9·003	9	−169	− 68	+0·2894	13·6
17	3·375	8·889	16·741	9·315	9	− 46	−103	·2922	13·7
18	3·481	8·868	16·594	9·625	9	+ 93	−111	·2949	13·7
19	3·581	8·874	16·443	9·932	9	+216	− 92	·2977	13·8
20	3·665	8·899	16·287	10·237	9	+297	− 54	·3004	13·9
21	− 3·729	8·933	−16·126	−10·539	9	+326	− 6	+0·3031	13·9
22	3·772	8·966	15·960	10·838	9	+302	+ 41	·3059	14·0
23	3·798	8·989	15·789	11·134	9	+233	+ 78	·3086	14·1
24	3·814	8·996	15·614	11·427	9	+137	+100	·3113	14·1
25	3·825	8·984	15·433	11·716	9	+ 28	+102	·3141	14·2
26	− 3·839	8·955	−15·248	−12·002	− 9	− 75	+ 88	+0·3168	14·3
27	3·862	8·910	15·059	12·284	9	−156	+ 58	·3196	14·3
28	3·899	8·854	14·864	12·562	10	−205	+ 18	·3223	14·4
29	3·950	8·797	14·666	12·836	10	−218	24	·3250	14·5
30	4·017	8·743	14·463	13·106	9	−194	− 62	·3278	14·5
May 1	+ 4·096	− 8·701	−14·256	−13·372	9	−140	− 89	+0·3305	14·6
2	4·183	8·672	14·045	13·633	9	− 67	−102	·3333	14·7
3	4·274	8·661	13·830	13·890	9	+ 12	− 97	·3360	14·7
4	4·360	8·669	13·611	14·143	9	+ 81	− 74	·3387	14·8
5	4·438	8·689	13·388	14·391	9	+126	− 38	·3415	14·9
6	+ 4·503	−8·720	−13·162	−14·634	9	+136	+ 9	+0·3442	14·9
7	4·550	8·752	12·933	14·873	9	+103	+ 57	·3469	15·0
→ 8	4·582	<u>8·773</u>	12·700	15·107	9	+ 28	+ 94	·3497	15·1
→ 9	4·603	<u>8·775</u>	12·464	15·337	9	− 75	+112	·3524	15·1
10	4·624	8·750	12·225	15·562	9	−181	+102	·3552	15·2
11	+ 4·657	8·697	−11·984	−15·783	− 9	−258	+ 65	+0·3579	15·3
12	4·712	8·627	11·739	15·999	9	−278	+ 10	·3606	15·3
13	4·797	8·552	11·492	16·211	9	−228	− 49	·3634	15·4
14	4·906	8·491	11·241	16·419	9	−116	− 95	·3661	15·5
15	5·030	8·457	10·988	16·623	9	+ 30	−114	·3688	15·5
16	+ 5·152	−8·453	−10·731	−16·823	9	+173	−104	+0·3716	15·6
17	+ 5·261	−8·473	−10·472	−17·018	8	+280	− 69	+0·3743	15·7

FIGURE 16 −"AMERICAN EPHEMERIS"

5.2 Reduction From Mean Place Data

The "Apparent Place of Fundamental Stars" and the "American Ephemeris" give the mean place of a star at the current BNY in the heliocentric mean equator-equinox coordinate system of the current BNY. This is the starting point, and the steps are as follows:

5.2.1 Summary of Computation Procedure

1. Apply proper motion to find the star mean place at the desired time.

2. Compute the unit vector $\overline{u}_c$ of the star's place.

3. Find the earth's position (from the Tables of the Sun) in ecliptic coordinates.

4. Use the constant of aberration κ to find the aberration components in ecliptic coordinates.

5. Transform the results of (3) and (4) into mean equator-equinox coordinates.

6. Apply the formula for the apparent place,

$$\overline{u}_e = \text{unit}\left[\overline{u}_c - \pi\overline{R} + \frac{\overline{V}_f^*}{c}\right] \quad .$$

5.2.2 Input

1. α_0 = catalog mean right ascension of the star in hours, minutes, and seconds.

2. δ_0 = catalog mean declination of the star in degrees, minutes, and seconds.

3. μ_α = annual proper motion of the star in right ascension expressed in hour seconds.

4. μ_δ = annual proper motion of the star in declination expressed in arc seconds.

5. π = stellar parallax in arc seconds.

6. κ = constant of aberration in arc seconds.

7. d_e = date of event in years, months, days and fractional part thereof of U.T.

8. d_p = date specified to the nearest integer day prior to d_e in years, months, and days of E.T.

9. d_{BNY} = date of the nearest BNY in years, months, days and fractional parts thereof of U.T.

10. λ_{oc} = true longitude of the sun from the mean equinox of the BNY in degrees, minutes, and seconds at d_p.

11. R_c = magnitude of the radius vector to the sun in astronomical units at d_p.

12. $\Delta_{\frac{1}{2}}'$ and $\Delta_{\frac{1}{2}}'$ = the first difference for λ_{oc} and R_c, respectively, given in arc seconds and astronomical units.

5.2.3 Input Data for Sample Computation

The mean place of α Tauri taken from the "Apparent Places of Stars" (**p.** 73, Figure 17) at d_{BNY} = 1.283 January 1968 U.T. is

$$\alpha_o = 4^h\ 34^m\ 04\overset{s}{.}892$$

$$\delta_o = 16°\ 26'\ 46\overset{"}{.}97$$

Alternately, the mean place of α Tauri taken from page 285 (Figure 18) of the "American Ephemeris" at d_{BNY} = 1.283 January 1968 is

$$\alpha_o = 4^h\ 34^m\ 04\overset{s}{.}9$$

$$\delta_o = 16°\ 26'\ 47"$$

The proper motion of α Tauri from "SAO Star Catalog" (Figure 19) is

$$\mu_\alpha = 0\overset{s}{.}0045$$

$$\mu_\delta = -0\overset{"}{.}189$$

α Tauri is located in the "SAO Star Catalog" by its approximate right ascension, magnitude (1.1 for α Tauri), and F K-4 catalog number (168), given in the APFS.

The parallax of α Tauri from the "General Catalogue of Trigonometric Parallaxes" (Figure 20) is

$$\pi = 0\overset{"}{.}048$$

α Tauri is located in the catalog by its approximate right ascension and magnitude. α Tauri is star number 1014 in this catalog.

The radius vector to the sun, the longitude of the sun, and their corresponding first differences are obtained from Page 22 and 23 (Figures 21 and 22) of the "American Ephemeris" at d_p = May 8, 1968 E.T. They are as follows

$$\lambda_{oc} = 47° \; 27' \; 51\overset{''}{.}2$$

$$\Delta'_{\frac{1}{2}} = 3479\overset{''}{.}9$$

$$R_c = 1.0094301$$

$$\Delta'_{\frac{1}{2}} = 0.0002288$$

$$d_e = 8\overset{d}{.}7333 \; \text{May 1968 U.T.}$$

and the constant (κ) of aberration (see "Supplement to the A.E. 1968) is

$$\kappa = 20\overset{''}{.}496$$

5.2.4 Sample Computation

The sample computation is given twice using first the mean place star data of the APFS and second that of the American Ephemeris. For the APFS star data, the computation proceeds as follows

1. Compute the star mean place at the event time, d_e

$$\alpha = \alpha_0 + \frac{(d_e - d_{BNY})}{365.242} \mu_\alpha = 4^h \; 34^m \; 04\overset{s}{.}891$$

and

$$\delta = \delta_0 + \frac{(d_e - d_{BNY})}{365.242} \mu_\delta = 16° \; 26' \; 46\overset{''}{.}996$$

2. Compute the components of the unit vector $\bar{u}_c$

$$X_c = \cos\delta\cos\alpha = 0.35119674$$

$$Y_c = \cos\delta\sin\alpha = 0.89247209$$

$$Z_c = \sin\delta \quad\quad = 0.28311550$$

3. Find the sun's longitude λ_0 at the event time d_e. Since the tables are in ephemeris time (E.T.), one determines the event time in E.T. by

$$d_e' = d_e + \Delta T$$

where for 1968, $\Delta T = 0\overset{d}{.}00004$

$$\lambda_0 = \lambda_{oc} + (d_e' - d_p) \, \Delta_{\frac{1}{2}}'$$

$$\lambda_0 = 170871\overset{''}{.}2 + (.7337)(3479.9)$$

$$\lambda_0 = 173424\overset{''}{.}20 = 48\overset{\circ}{.}173888$$

4. Find the longitude of the earth (λ_e) at the event time d_e

$$\lambda_e = \lambda_0 - 180°$$

$$\lambda_e = -131\overset{\circ}{.}8266111$$

One should note that had the event occurred before the BNY, λ_e would be measured from the equinox of the preceding BNY. The annual precession, computed from equation 28 of the text, along the ecliptic would have to be subtracted from λ_e.

5. Find the magnitude of the radius vector in astronomical units from the sun to the earth

$$R = R_c + (d_e' - d_p) \, \Delta_{\frac{1}{2}}'$$

$$R = 1.0094301 + (.7337)(0.0002288)$$

$$R = 1.00959797 \text{ a.u.}$$

6. Find the components of the earth's position vector in ecliptic coordinates

$$X_{ecl.} = R \cos\lambda_e = -0.67326612 \text{ a.u.}$$

$$Y_{ecl.} = R \sin\lambda_e = -0.75233018 \text{ a.u.}$$

$$Z_{ecl.} \qquad\qquad = 0.0$$

7. Compute the components of the aberration vector, $\dfrac{\overline{V}_{f*}}{c}$, in ecliptic coordinates. This vector leads $\overline{R}$ by 90°, hence

$$\frac{\dot{X}_f}{c} ecl = -\kappa \sin\lambda_e = 7.4045847 \times 10^{-5}$$

$$\frac{\dot{Y}_f}{c} ecl = \kappa \cos\lambda_e = -6.6264203 \times 10^{-5}$$

$$\frac{\dot{Z}_f}{c} ecl = 0$$

8. Compute the mean obliquity using a formula given on page 98 of the "Explanatory Supplement to the Ephemeris"

$$\epsilon_o = 23\overset{\circ}{.}452294 - 0\overset{\circ}{.}0035626D$$

$$- 0\overset{\circ}{.}000000123D^2 + 0\overset{\circ}{.}0000000103D^3$$

where

$$D = 10^{-4}(d_e - d_{Jan\ 0\overset{d}{.}5,\ 1900})$$

d_e and $d_{Jan\ 0\overset{d}{.}5,\ 1900}$ are converted into Julian Dates

$$\text{J.D. of } d_{Jan\ 0.5\ 1900} = 2415020.0$$

$$\text{J.D. of } d_e \qquad = 2439985.233$$

The mean obliquity is

$$\epsilon_{o} = 23\overset{\circ}{.}443410$$

9.   Transform the components of $\overline{R}$ and $\dfrac{\overline{V}_{f*}}{c}$ from the ecliptic coordinate system of the nearest BNY to the mean equatorial coordinate system of the nearest BNY by

$$\overline{R}_{eq.} = [T]\ \overline{R}_{ecl.}$$

and

$$\frac{\overline{V}_{f*}}{c}\ eq. = [T]\ \frac{\overline{V}_{f*}}{c}\ ecl.$$

where

$$[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.91745347 & -0.39784309 \\ 0 & 0.39784309 & 0.91745347 \end{bmatrix}$$

Then

$$X_{eq.} = -0.67326612\ a.u.$$

$$Y_{eq.} = -0.69022793\ a.u.$$

$$Z_{eq.} = -.029930937\ a.u.$$

and

$$\frac{\dot{X}_f}{c}\ eq. = 7.4045847 \times 10^{-5}$$

$$\frac{\dot{Y}_f}{c}\ eq. = -6.0794323 \times 10^{-5}$$

$$\frac{\dot{Z}_f}{c}\ eq. = -2.6362755 \times 10^{-5}$$

Multiply the components of $\overline{R}_{eq.}$ by π (π must first be converted into radians).

$$\pi\ X_{eq.} = 1.5667495 \times 10^{-7}$$

$$\pi\ Y_{eq.} = 1.6062211 \times 10^{-7}$$

$$\pi\ Z_{eq.} = 6.9651922 \times 10^{-8}$$

10. Compute the components of the unit vector for the apparent place from the earth by

$$\overline{u}_e = \text{unit}\left[\overline{u}_c - \pi\ \overline{R} + \frac{\overline{V}_{f*}}{c}\right]$$

They are

$$X = 0.35128342$$

$$Y = 0.89244316$$

$$Z = 0.28309926$$

The computation using the mean place star data of the American Ephemeris proceeds as follows

1. Compute the star mean place at the event time d_e

$$\alpha = \alpha_0 + \frac{(d_e - d_{BNY})}{365.242}\ \mu_\alpha = 4^h\ 34^m\ 04^s.899$$

$$\delta = \delta_0 + \frac{(d_e - d_{BNY})}{365.242}\ \mu_\delta = 16°\ 26'\ 46''.997$$

2. Compute the components of the unit vector u_c

$$X_c = \cos\delta\cos\alpha = .35119620$$

$$Y_c = \cos\delta\sin\alpha = .89247226$$

$$Z_c = \sin\delta\qquad\ \ = .28311564$$

3. The computations of Steps 3-9 are exactly the same as those of the first computation. Hence, they are not repeated here.

4. Compute the components of the unit vector for the apparent place by

$$\bar{u}_e = \text{unit}\left[\bar{u}_c - \pi \bar{R} + \frac{\bar{V}_{f*}}{c}\right]$$

They are

$$X = 0.35128287$$

$$Y = 0.89244333$$

$$Z = 0.28309941$$

APPARENT PLACES OF STARS, 1968

AT UPPER TRANSIT AT GREENWICH

No.	168		170		169		172	
Name	α Tauri (Aldebaran)		υ² Eridani		μ Eridani		53 Eridani*	
Mag. Spect.	1.06 K5		3.88 K0		4.12 B2		3.98 K0	
U.T.	R.A.	Dec.	R.A.	Dec.	R.A.	Dec.	R.A.	Dec.
	h m 4 34	+16 26	h m 4 34	-30 37	h m 4 34	-3 24	h m 4 36	-14 21
I -6.1	05.600 +62 (19)	55.06 -19	19.514 -36 (11)	35.30 -236	43.866 +8 (49)	54.58 -124	43.690 -3 (38)	51.84 -176
I 3.9	05.619 -23	54.87 -19	19.479 -79	37.66 -206	43.874 -32	55.82 -111	43.687 -44	53.60 -157
I 13.9	05.596 -43	54.68 -19	19.400 -118	39.72 -174	43.842 -69	56.93 -96	43.643 -82	55.17 -133
I 23.9	05.533 -100	54.49 -19	19.282 -156	41.46 -137	43.773 -104	57.89 -79	43.561 -116	56.50 -108
II 2.8	05.433 -131	54.30 -19	19.126 -185	42.83 -93	43.669 -133	58.68 -59	43.445 -145	57.58 -78
II 12.8	05.302 -151	54.11 -21	18.941 -209	43.76 -53	43.536 -151	59.27 -41	43.300 -164	58.36 -49
II 22.8	05.151 -145	53.90 -21	18.738 -217	44.29 -10	43.385 -146	59.68 -20	43.136 -178	58.85 -18
II 3.7	04.986 -166	53.69 -20	18.521 -217	44.39 +36	43.220 -166	59.88 +2	42.958 -178	59.03 +13
II 13.7	04.820 -157	53.49 -19	18.304 -207	44.03 +75	43.054 -157	59.86 +21	42.780 -170	58.90 +43
II 23.7	04.663 -139	53.30 -16	18.097 -188	43.28 +117	42.897 -141	59.65 +43	42.610 -154	58.47 +73
IV 2.7	04.524 -108	53.14 -11	17.909 -159	42.11 +156	42.756 -113	59.22 +66	42.456 -126	57.74 +104
IV 12.6	04.416 -74	53.03 -3	17.750 -123	40.55 +189	42.643 -81	58.56 +85	42.330 -93	56.70 +129
IV 22.6	04.342 -31	53.00 +7	17.627 -62	38.66 +221	42.562 -41	57.71 +106	42.237 -54	55.41 +156
V 2.6	04.311 +15	53.07 +19	17.545 -33	36.45 +248	42.521 +3	56.65 +126	42.183 -10	53.85 +179
V 12.6	04.326 +60	53.26 +31	17.512 -13	33.97 +268	42.524 +45	55.39 +144	42.173 +33	52.06 +197
V 22.5	04.386 +108	53.57 +45	17.525 +63	31.29 +286	42.569 +91	53.95 +159	42.206 +80	50.09 +215
VI 1.5	04.494 +153	54.02 +59	17.588 +133	28.43 +294	42.660 +134	52.36 +171	42.286 +123	47.94 +225
VI 11.5	04.647 +192	54.61 +71	17.701 +155	25.49 +295	42.794 +171	50.65 +179	42.409 +162	45.69 +231
VI 21.4	04.839 +229	55.32 +83	17.856 +199	22.54 +292	42.965 +208	48.86 +184	42.571 +200	43.38 +232
VII 1.4	05.068 +259	56.15 +91	18.055 +234	19.62 +276	43.173 +237	47.02 +183	42.771 +231	41.06 +225
VII 11.4	05.327 +282	57.06 +97	18.289 +264	16.86 +256	43.410 +260	45.19 +176	43.002 +256	38.81 +212
VII 21.4	05.609 +301	58.03 +100	18.553 +270	14.30 +228	43.670 +281	43.43 +164	43.258 +278	36.69 +194
VII 31.3	05.910 +313	59.03 +99	18.843 +306	12.02 +190	43.951 +291	41.79 +147	43.536 +291	34.75 +168
VIII 10.3	06.223 +318	60.02 +95	19.149 +317	10.12 +151	44.242 +299	40.32 +126	43.827 +298	33.07 +138
VIII 20.3	06.541 +321	60.97 +88	19.466 +324	08.61 +102	44.541 +302	39.06 +100	44.125 +304	31.69 +102
VIII 30.3	06.862 +317	61.85 +77	19.790 +320	07.59 +51	44.843 +298	38.06 +69	44.429 +299	30.67 +62
IX 9.2	07.179 +309	62.62 +66	20.110 +314	07.08 +0	45.141 +291	37.37 +39	44.728 +294	30.05 +23
IX 19.2	07.488 +300	63.27 +51	20.424 +301	07.08 -56	45.432 +282	36.98 +5	45.022 +283	29.82 -20
IX 29.2	07.788 +283	63.78 +36	20.725 +281	07.63 -105	45.714 +266	36.93 -27	45.305 +266	30.02 -61
X 9.1	08.071 +268	64.14 +24	21.006 +261	08.68 -151	45.979 +250	37.20 -56	45.571 +250	30.63 -98
X 19.1	08.339 +247	64.38 +10	21.267 +232	10.19 -194	46.229 +228	37.76 -83	45.821 +227	31.61 -132
X 29.1	08.586 +221	64.48 +0	21.499 +200	12.13 -226	46.457 +204	38.59 -106	46.048 +200	32.93 -160
XI 8.1	08.807 +196	64.48 -9	21.699 +166	14.39 -251	46.661 +177	39.65 -122	46.248 +172	34.53 -180
XI 18.0	09.003 +162	64.39 -15	21.865 +125	16.90 -267	46.838 +145	40.87 -134	46.420 +138	36.33 -194
XI 28.0	09.165 +127	64.24 -19	21.990 +83	19.57 -269	46.983 +110	42.21 -138	46.558 +102	38.27 -198
XII 7.9	09.292 +87	64.05 -20	22.073 +40	22.26 -265	47.093 +75	43.59 -137	46.660 +65	40.25 -196
XII 17.9	09.381 +46	63.85 -22	22.113 -7	24.91 -251	47.168 +33	44.96 -133	46.725 +22	42.21 -188
XII 27.9	09.427 +5	63.63 -21	22.106 -51	27.42 -226	47.201 -6	46.29 -121	46.747 -18	44.09 -170
XII 37.9	09.432 -37	63.42 -21	22.055 -94	29.68 -198	47.195 -45	47.50 -109	46.729 -57	45.79 -152
Mean Place	04.892	46.97	18.314	36.92	43.070	59.85	42.777	55.66
sec δ, tan δ	+1.043	+0.295	+1.162	-0.592	+1.002	-0.060	+1.032	-0.256
da(ψ),dδ(ψ)	+0.068	+0.15	+0.047	+0.15	+0.060	+0.14	+0.055	+0.14
da(ε),dδ(ε)	-0.007	+0.93	+0.014	+0.93	+0.001	+0.93	+0.006	+0.93
Dble. Trans.	November 29		November 29		November 29		November 30	

FIGURE 17 – "APPARENT PLACES OF FUNDAMENTAL STARS"

MEAN PLACES OF STARS, 1968.0

FOR JANUARY 1ᵈ.283

Name	Mag.	Sp.	Right Ascension	Declination	Name	Mag.	Sp.	Right Ascension	Declination
			h m s	° ′ ″				h m s	° ′ ″
16 Per	4.3	F0	2 48 33.3	+38 11 17	γ Hyi	3.2	M0	3 47 43.0	−74 20 15
17 Per	4.7	K5	2 49 32.2	+34 55 45	g Eri	4.2	K0	3 48 15.3	−36 17 47
ν Hyi	4.7	K2	2 50 39.5	−75 11 52	ζ Per	2.9	B1	3 52 06.9	+31 47 24
τ Per	4.1	G0, A5	2 51 58.6	+52 37 58	ε Per	3.0	B1	3 55 42.0	+39 55 09
η Eri	4.0	K0	2 54 51.7	− 9 01 28	γ Eri	3.2	K5	3 56 32.1	−13 35 54
π Per	4.6	A2	2 56 42.4	+39 32 09	ξ Per	4.0	Oe5	3 56 53.0	+35 42 02
θ Eri	3.4	A2	2 57 02.8	−40 25 56	δ Ret	4.4	M0	3 58 14.1	−61 29 25
ε Ari	4.6	A2	2 57 22.6	+21 12 48	36 Eri	4.7	A0p	3 58 33.5	−24 06 22
λ Cet	4.7	B5	2 57 59.8	+ 8 46 51	λ Tau	3.9	B3	3 58 54.3	+12 24 04
α Cet	2.8	M0	3 00 36.2	+ 3 57 55	γ Ret	4.5	M5	4 00 25.7	−62 14 54
τ³ Eri	4.2	A3	3 00 58.8	−23 44 56	ν Tau	3.9	A0	4 01 27.1	+ 5 54 06
γ Per	3.1	F5, A3	3 02 28.0	+53 22 57	37 Tau	4.5	K0	4 02 48.0	+21 59 45
ρ Per	3–4	M3	3 03 07.1	+38 43 03	λ Per	4.3	A0	4 04 11.5	+50 15 58
β Per	2–3	B8	3 06 04.7	+40 50 01	48 Per	4.0	B3p	4 06 19.8	+47 37 44
ι Per	4.2	G0	3 06 44.8	+49 29 33	o¹ Eri	4.1	F2	4 10 18.1	− 6 55 11
κ Per	4.0	K0	3 07 19.7	+44 44 15	μ Per	4.3	G0	4 12 32.4	+48 19 47
δ Ari	4.5	K0	3 09 47.7	+19 36 25	α Hor	3.8	K0	4 12 56.4	−42 22 21
α For	3.9	F8	3 10 42.6	−29 06 45	40 Eri	4.5	G5	4 13 47.9	− 7 42 05
16 Eri	3.9	M3	3 18 05.5	−21 52 24	μ Tau	4.3	B3	4 13 47.7	+ 8 48 48
+28°516	4.7	K5	3 18 23.9	+28 56 01	α Ret	3.4	G5	4 14 00.5	−62 33 13
82 G. Eri	4.3	G5	3 18 39.1	−43 11 29	γ Dor	4.4	F5	4 15 11.2	−51 34 00
α Per	1.9	F5	3 22 01.7	+49 44 56	ε Ret	4.4	K2	4 15 55.6	−59 22 43
o Tau	3.8	G5	3 23 05.3	+ 8 55 03	b Per	4.6	A2	4 15 49.8	+50 13 07
ξ Tau	3.7	B8	3 25 25.9	+ 9 37 21	41 Eri	3.6	B9	4 16 40.9	−33 52 32
2 H. Cam	4.4	B9p	3 26 27.7	+59 49 50	γ Tau	3.9	K0	4 17 58.2	+15 33 06
34 Per	4.7	B5	3 27 04.2	+49 23 58	δ Tau	3.9	K0	4 21 05.2	+17 28 08
σ Per	4.5	K0	3 28 18.5	+47 53 11	43 Eri	4.1	K5	4 22 50.0	−34 05 25
5 Tau	4.3	K0	3 29 06.2	+12 49 41	κ Tau	4.4	A3	4 23 27.5	+22 13 19
ε Eri	3.8	K0	3 31 25.3	− 9 33 56	68 Tau	4.2	A2	4 23 38.1	+17 51 23
τ⁵ Eri	4.3	B8	3 32 22.4	−21 44 21	υ Tau	4.4	A5	4 24 23.4	+22 44 33
ψ Per	4.3	B5p	3 34 12.4	+48 05 16	71 Tau	4.6	A5	4 24 31.2	+15 32 49
10 Tau	4.4	G5	3 35 14.2	+ 0 18 04	77 Tau	4.0	K0	4 26 44.6	+15 53 33
y Eri	4.6	K0	3 35 56.7	−40 22 44	● Tau	3.6	K0	4 26 44.7	+19 06 39
δ Per	3.1	B5	3 40 38.3	+47 41 13	θ² Tau	3.6	F0	4 26 49.9	+15 48 05
h Eri	4.6	K2	3 41 38.8	−37 24 49	ρ Tau	4.7	A5	4 32 01.8	+14 46 43
δ Eri	3.7	K0	3 41 42.8	− 9 52 15	50 Eri	4.6	K0	4 32 15.2	−29 49 49
o Per	3.9	B1	3 42 18.4	+32 11 18	α Dor	3.5	A0p	4 33 18.2	−55 06 38
17 Tau	3.8	B5p	3 42 58.3	+24 00 51	88 Tau	4.4	A3	4 33 53.6	+10 05 47
ν Per	3.9	F5	3 43 00.8	+42 28 45	α Tau	1.1	K5	4 34 04.9	+16 26 47 ←
19 Tau	4.4	B5	3 43 17.9	+24 22 05	υ Eri	3.9	K0	4 34 18.3	−30 37 37
β Ret	3.8	K0	3 43 47.5	−64 54 27	58 Per	4.5	K0, A3	4 34 28.0	+41 12 03
20 Tau	4.0	B5	3 43 55.1	+24 16 09	ν Eri	4.1	B2	4 34 43.1	− 3 25 00
23 Tau	4.2	B5	3 44 25.4	+23 51 00	90 Tau	4.3	A3	4 36 22.0	+12 26 53
π Eri	4.6	M2	3 44 37.6	−12 12 03	53 Eri	4.0	K0	4 36 42.8	−14 21 56
τ⁶ Eri	4.3	F8	3 45 28.2	−23 20 36	54 Eri	4.5	M4	4 39 02.4	−19 43 55
η Tau	3.0	B5p	3 45 34.7	+24 00 27	α Cae	4.5	F2	4 39 31.7	−41 55 27
+65°369	4.7	M1	3 46 34.2	+65 25 45	τ Tau	4.3	B5	4 40 19.3	+22 53 49
γ Cam	4.7	A0	3 46 57.0	+71 14 10	μ Eri	4.2	B5	4 43 54.0	− 3 18 43
27 Tau	3.8	B8	3 47 15.3	+23 57 25	π³ Ori	3.3	F8	4 48 06.1	+ 6 54 25

FIGURE 18 – "AMERICAN EPHEMERIS"

FIGURE 19 -"SMITHSONIAN ASTROPHYSICAL OBSERVATORY STAR CATALOG"

42   YALE UNIVERSITY OBSERVATORY

4^h

No	DM	α (1900) δ	Mag.	Sp.	HD	μ cat	μ_α	μ_δ	Absolute π
1001	-44°1590	27.4^m -44°13'	11.2	K			+."15	+ ."09	+."009 ± 13
1002	+55 900	27.9 +55 13	8.6	K1		Ci 20, 296	+.57	- .28	+ 56 7
1003	+14 720	28.2 +14 38	4.8	A5	28910	GC 5558	+.102	- .028	+ 22 6
1004	+ 5 678	28.5 + 5 11	8.0	K1	28946	Ci 18, 592	-.12	- .26	+ 41 13
1005	-49 1366	29.1 -49 33	8.7	G0	29029		-.01	+ .35	+ 21 14
1006		29.3 -43 14	11.9				+.19	+ .17	+ 22 ± 12
1007	- 8 887	29.4 - 8 26	5.4	Ma	29064	GC 5576	-.027	+ .007	0 9
1008	- 9 930	29.4 - 9 11	5.5	K2	29065	GC 5577	-.037	- .109	+ 7 5
1009	-30 1883	29.6 -29 58	4.6	K0	29085	GC 5572	-.107	- .274	+ 18 8
1010	+52 857	29.8 +52 42	8.5	K6	232979	Ci 18, 594	+.27	- .46	+ 91 7
1011	+40 1000	29.8 +41 4	4.5		29094	GC 5609	-.011	- .018	+ 20 ± 5
1012		30.0 +67 24	11.2	K5			+.24	- .28	+ 12 9
1013	-68 268	30.1 -68 6	7.8	G0	29137	GC 5544	+.212	+ .420	+ 14 8
1014	+16 629	30.2 +16 18	1.1	K5	29139	GC 5605	+.069	- .190	+ 48 4
1015	+ 9 607	30.2 + 9 57	4.4	A3	29140	GC 5599	+.056	- .045	+ 30 5
1016	+18 661	31.4 +18 20	var	G0	29260	GC 5621	-.014	- .007	+ 14 ± 11
1017	-30 1901	31.7 -30 46	3.9	K0	29291	GC 5614	-.054	- .011	- 18 12
1018	-55 663	31.8 -55 15	3.5	A0p	29305	GC 5600	+.051	- .001	+ 11 11
1019	+53 794	32.0 +53 17	5.4	F0	29316	GC 5659	+.052	- .092	+ 18 8
1020	+76 174	32.1 +76 25	6.5	F5	29329	GC 5711	+.073	- .132	+ 14 10
1021	+53 796	32.5 +53 17	9.3 10.3	A3	29362				- 3 ± 11
1022	+12 618	32.6 +12 19	4.3	A3	29388	GC 5645	+.101	- .012	+ 18 5
1023	- 2 963	32.6 - 2 40	5.3	A5	29391	GC 5635	+.042	- .058	+ 26 10
1024	+18 667	32.7 +18 52	9.3	A2	29402				+ 3 12
1025	+66 343	32.8 +66 32	8.9	G5	29400	GC 5688	+.377	+ .072	+ 20 11
1026	-11 916	33.0 -11 14	10.9	M0			-.22	+ .20	+ 93 ± 11
1027	-46 1466	33.5 -46 42	11.8				+.11	+ .18	+ 28 13
1028	-14 933	33.6 -14 30	4.0	K0	29503	GC 5657	-.073	- .158	+ 36 7
1029	-42 1571	33.6 -42 52	11.3	K7			+.14	+ .01	+ 15 12
1030	+ 9 621	34.2 + 9 41	8.8	K2		Ci 18, 601	-.02	- .36	+ 43 8
1031	-12 955	34.2 -12 19	5.0	A2	29573	GC 5669	-.054	- .009	+ 34 ± 11
1032	+41 931	34.5 +41 56	7.3	G2	29587	GC 5692	+.546	- .417	+ 24 6
1033	+56 964	34.6 +57 1	8.0	F8	29599		-.01	+ .03	+ 2 13
1034	-14 936	34.7 -14 33	5.6	G5	29613	GC 5678	+.122	- .125	0 13
1035	+37 954	35.0 +38 5	5.8	F5	29645	GC 5701	+.241	- .098	+ 23 9
1036	-40 1499	35.1 -40 23	9.1	F8	29666		+.22	+ .17	+ 20 ± 11
1037	+75 189	35.4 +75 46	6.0	F0	29678	GC 5774	+.042	- .131	+ 18 10
1038		35.4 +22 43	13.0	K2		Ci 20, 301	+.37	- .57	+ 11 9
1039	+20 802	35.5 +20 43	9.0	K3	29697	GC 5699	-.230	- .262	+ 79 10
1040	+69 271	35.7 +69 54	8.8	K0	29713	Ci 18, 607	+.11	- .03	+ 29 10
1041	+43 1043	35.8 +43 10	5.2	A0	29722	GC 5719	+.043	- .051	+ 16 ± 6
1042	-19 988	36.1 -19 52	4.5	Ma	29755	GC 5695	+.023	- .094	+ 2 10
1043	+22 739	36.2 +22 46	4.3	B5	29763	GC 5716	+.004	- .016	+ 8 14
1044	+22 737	36.2 +22 45	7.8	A1		GC 5715	-.011	- .027	- 57 12
1045	+69 272	36.4 +70 4	8.4	K0	29775		+.01	- .05	+ 9 9
1046	+18 683	37.0 +18 47	10.0	M3		Ci 20, 303	+.71	-1.05	+ 98 ± 6
1047	+19 764	37.1 +20 6	9.2 9.5	K2 K2					+ 8 10
1048	-42 1587	37.3 -42 3	4.5	F2	29875	GC 5708	-.148	- .080	+ 38 8
1049	+27 688	37.4 +27 30	8.0	K3	29883	GC 5747	+.065	- .241	+ 45 5
1050	-65 361	37.6 -65 39	9.6	G0	29907	Ci 20, 302	+.67	+1.31	+ 3 8

FIGURE 20-"GENERAL CATALOGUE OF TRIGONOMETRIC PARALLAXES"

22 λ_{oc}

SUN, 1968
FOR 0ʰ EPHEMERIS TIME

Date	Longitude Mean Equinox of 1968·0	Redn to App. Long.	Latitude Ecliptic of 1968·0	1950·0	Date	Hor. Par.	Prec. in Long.	Nutation in Long.	Obl. of Ecliptic 23° 26′
Apr. 1	11 20 00·6	14·1	-0·01	2·43	+0·04	8·80	+12·486	6·112	45·428
2	12 19 12·4 3551·8	14·0	·11	2·46	·15	8·80	12·624	6·139	45·380
3	13 18 21·0 3549·5	13·8	·20	2·51	·24	8·80	12·761	6·129	45·337
4	14 17 29·2 3547·3	13·7	·27	2·58	·31	8·80	12·899	6·091	45·305
5	15 16 34·2 3545·0	13·5	·31	2·68	·35	8·79	13·037	6·035	45·289
	3542·6								
6	16 15 36·8 3540·4	13·3	+0·33	2·80	+0·37	8·79	+13·174	- 5·975	45·291
7	17 14 37·2 3538·1	13·1	·31	2·95	·36	8·79	13·312	5·927	45·311
8	18 13 35·3 3535·8	12·9	·26	3·13	·31	8·79	13·450	5·907	45·343
9	19 12 31·1 3533·5	12·8	·19	3·34	·24	8·79	13·587	5·927	45·384
10	20 11 24·6 3531·2	12·7	·09	3·57	·14	8·78	13·725	5·993	45·419
11	21 10 15·8 3529·1	12·7	0·03	3·82	+0·03	8·78	+13·863	- 6·098	45·440
12	22 09 04·9 3527·0	12·6	·17	4·09	·11	8·78	14·000	6·223	45·435
13	23 07 51·9 3525·0	12·6	·31	4·36	·25	8·78	14·138	6·335	45·402
14	24 06 36·9 3523·1	12·5	·45	4·62	·39	8·77	14·276	6·405	45·343
15	25 05 20·0 3521·3	12·4	·59	4·88	·52	8·77	14·413	6·410	45·270
16	26 04 01·3 3519·5	-12·2	-0·70	-5·12	-0·63	8·77	+14·551	- 6·347	45·204
17	27 02 40·8 3517·8	11·9	·79	5·34	·72	8·77	14·688	6·235	45·156
18	28 01 18·6 3516·2	11·7	·85	5·51	·77	8·76	14·826	6·106	45·133
19	28 59 54·8 3514·5	11·4	·88	5·66	·80	8·76	14·964	5·992	45·138
20	29 58 29·3 3512·9	11·2	·87	5·77	·79	8·76	15·101	5·919	45·162
21	30 57 02·2 3511·3	-11·0	-0·83	- 5·85	0·75	8·76	+15·239	- 5·897	45·195
22	31 55 33·5 3509·6	10·9	·76	5·90	·67	8·75	15·377	5·926	45·226
23	32 54 03·1 3508·0	10·8	·67	5·91	·58	8·75	15·514	5·999	45·248
24	33 52 31·1 3506·2	10·8	·56	5·92	·46	8·75	15·652	6·097	45·254
25	34 50 57·3 3504·6	10·8	·43	5·90	·33	8·75	15·790	6·207	45·240
26	35 49 21·9 3502·8	10·7	-0·30	- 5·88	-0·20	8·74	+15·927	- 6·310	45·210
27	36 47 44·7 3501·0	10·7	·17	5·86	·07	8·74	16·065	6·390	45·164
28	37 46 05·7 3499·1	10·6	·05	5·84	+ ·06	8·74	16·202	6·436	45·107
29	38 44 24·8 3497·3	10·4	+ ·07	5·83	·17	8·74	16·340	6·444	45·048
30	39 42 42·1 3495·5	10·3	·16	5·84	·27	8·73	16·478	6·415	44·993
May 1	40 40 57·6 3493·5	-10·1	+0·24	5·87	+0·35	8·73	+16·615	- 6·354	44·950
2	41 39 11·1 3491·5	9·8	·29	5·91	·40	8·73	16·753	6·272	44·919
3	42 37 22·6 3489·7	9·6	·31	5·98	·43	8·73	16·891	6·183	44·907
4	43 35 32·3 3487·6	9·4	·31	6·08	·43	8·73	17·028	6·103	44·914
5	44 33 39·9 3485·8	9·2	·28	6·21	·40	8·72	17·166	6·045	44·933
6	45 31 45·7 3483·7	- 9·0	+0·21	-6·36	+0·34	8·72	+17·304	- 6·021	44·962
7	46 29 49·4 3481·8	8·9	·13	6·54	·25	8·72	17·441	6·039	44·993
➔ 8	47 27 51·2 3479·9	8·8	+ ·01	6·74	·14	8·72	17·579	6·097	45·013
9	48 25 51·1 3478·1	8·7	- ·12	6·95	+ ·02	8·72	17·717	6·182	45·013
10	49 23 49·2 3476·3	8·7	·25	7·17	·12	8·71	17·854	6·268	44·987
11	50 21 45·5 3474·6	- 8·6	-0·39	-7·39	0·25	8·71	+17·992	- 6·324	44·933
12	51 19 40·1 3473·0	8·5	·52	7·60	·38	8·71	18·129	6·322	44·862
13	52 17 33·1 3471·6	8·2	·64	7·80	·49	8·71	18·267	6·248	44·785
14	53 15 24·7 3470·1	8·0	·73	7·96	·58	8·71	18·405	6·111	44·723
15	54 13 14·8 3468·9	7·6	·80	8·10	·65	8·70	18·542	5·938	44·688
16	55 11 03·7 3467·7	- 7·3	-0·83	-8·21	-0·68	8·70	+18·680	- 5·768	44·682
17	56 08 51·4	- 7·1	-0·83	-8·28	-0·68	8·70	+18·818	- 5·631	44·701

To obtain the longitude referred to the mean equinox of 1950·0, subtract 15′ 04″·9.

FIGURE 21 – "AMERICAN EPHEMERIS"

SUN, 1968

FOR 0ʰ EPHEMERIS TIME

Date	Apparent Right Ascension		Apparent Declination		Radius Vector		Semi-diameter	Ephemeris Transit	
	h m s		° ′ ″				′ ″	h m s	s
Apr. 1	0 41 39.72	218.70	+ 4 28 59.2	+1388.0	0.999 4508	+2843	16 01.71	12 03 51.48	-17.80
2	0 45 18.42	218.81	4 52 07.2	1382.7	0.999 7351	2834	16 01.43	12 03 33.68	-17.68
3	0 48 57.23	218.94	5 15 09.9	1377.0	1.000 0185	2826	16 01.16	12 03 16.00	17.55
4	0 52 36.17	219.09	5 38 06.9	1371.0	.000 3011	2819	16 00.89	12 02 58.45	17.38
5	0 56 15.26	219.25	6 00 57.9	1364.6	.000 5830	2814	16 00.62	12 02 41.07	17.21
6	0 59 54.51	219.44	+ 6 23 42.5	+1357.9	1.000 8644	+2809	16 00.35	12 02 23.86	-17.02
7	1 03 33.95	219.63	6 46 20.4	1350.8	.001 1453	2807	16 00.08	12 02 06.84	16.82
8	1 07 13.58	219.85	7 08 51.2	1343.5	.001 4260	2807	15 59.81	12 01 50.02	16.58
9	1 10 53.43	220.08	7 31 14.7	1335.6	.001 7067	2809	15 59.54	12 01 33.44	16.34
10	1 14 33.51	220.34	7 53 30.3	1327.6	.001 9876	2814	15 59.27	12 01 17.10	16.08
11	1 18 13.85	220.61	+ 8 15 37.9	+1319.2	1.002 2690	+2819	15 59.00	12 01 01.02	-15.79
12	1 21 54.46	220.90	8 37 37.1	1310.6	.002 5509	2826	15 58.73	12 00 45.23	15.48
13	1 25 35.36	221.23	8 59 27.7	1301.5	.002 8335	2833	15 58.46	12 00 29.75	15.15
14	1 29 16.59	221.57	9 21 09.2	1292.4	.003 1168	2838	15 58.19	12 00 14.60	14.81
15	1 32 58.16	221.93	9 42 41.6	1282.7	.003 4006	2841	15 57.92	11 59 59.79	14.44
16	1 36 40.09	222.32	+10 04 04.3	+1273.0	1.003 6847	+2842	15 57.65	11 59 45.35	-14.05
17	1 40 22.41	222.71	10 25 17.3	1262.7	.003 9689	2838	15 57.38	11 59 31.30	13.65
18	1 44 05.12	223.12	10 46 20.0	1252.2	.004 2527	2832	15 57.11	11 59 17.65	13.23
19	1 47 48.24	223.54	11 07 12.2	1241.4	.004 5359	2820	15 56.84	11 59 04.42	13.23
20	1 51 31.78	223.98	11 27 53.6	1230.2	.004 8179	2804	15 56.57	11 58 51.62	12.80
21	1 55 15.76	224.42	+11 48 23.8	+1218.7	1.005 0983	+2785	15 56.30	11 58 39.27	12.35
22	1 59 00.18	224.89	12 08 42.5	1206.8	.005 3768	2763	15 56.04	11 58 27.38	11.89
23	2 02 45.07	225.36	12 28 49.3	1194.5	.005 6531	2737	15 55.78	11 58 15.95	11.43
24	2 06 30.43	225.84	12 48 43.8	1182.0	.005 9268	2709	15 55.52	11 58 05.00	10.95
25	2 10 16.27	226.33	13 08 25.8	1169.0	.006 1977	2680	15 55.26	11 57 54.54	10.46
26	2 14 02.60	226.83	+13 27 54.8	+1155.8	1.006 4657	+2647	15 55.01	11 57 44.57	9.97
27	2 17 49.43	227.34	13 47 10.6	1142.1	.006 7304	2615	15 54.75	11 57 35.10	-9.47
28	2 21 36.77	227.85	14 06 12.7	1128.2	.006 9919	2582	15 54.51	11 57 26.14	8.96
29	2 25 24.62	228.37	14 25 00.9	1113.9	.007 2501	2549	15 54.26	11 57 17.69	8.45
30	2 29 12.99	228.89	14 43 34.8	1099.4	.007 5050	2514	15 54.02	11 57 09.76	7.93
May 1	2 33 01.88	229.42	+15 01 54.2	+1084.3	1.007 7564	+2481	15 53.78	11 57 02.36	7.40
2	2 36 51.30	229.95	15 19 58.5	1069.1	.008 0045	2449	15 53.55	11 56 55.48	- 6.88
3	2 40 41.25	230.49	15 37 47.6	1053.4	.008 2494	2418	15 53.32	11 56 49.14	6.34
4	2 44 31.74	231.03	15 55 21.0	1037.5	.008 4912	2388	15 53.09	11 56 43.34	5.80
5	2 48 22.77	231.56	16 12 38.5	1021.3	.008 7300	2359	15 52.86	11 56 38.07	5.27
6	2 52 14.33	232.10	+16 29 39.8	+1004.7	1.008 9659	+2333	15 52.64	11 56 33.35	4.72
7	2 56 06.43	232.65	16 46 24.5	987.8	.009 1992	2309	15 52.42	11 56 29.17	- 4.18
→ 8	2 59 59.08	233.20	17 02 52.3	970.7	.009 4301	2288	15 52.20	11 56 25.54	3.63
9	3 03 52.28	233.75	17 19 03.0	953.3	.009 6589	2269	15 51.98	11 56 22.46	3.08
10	3 07 46.03	234.32	17 34 56.3	935.5	.009 8858	2253	15 51.77	11 56 19.95	2.51
11	3 11 40.35	234.88	+17 50 31.8	+ 917.6	1.010 1111	+2238	15 51.56	11 56 17.99	1.96
12	3 15 35.23	235.46	18 05 49.4	899.4	.010 3349	2224	15 51.35	11 56 16.60	- 1.39
13	3 19 30.69	236.04	18 20 48.8	880.9	.010 5573	2208	15 51.14	11 56 15.79	0.81
14	3 23 26.73	236.62	18 35 29.7	862.3	.010 7781	2191	15 50.93	11 56 15.55	- 0.24
15	3 27 23.35	237.20	18 49 52.0	843.2	.010 9972	2172	15 50.72	11 56 15.89	+ 0.34
16	3 31 20.55	237.77	+19 03 55.2	+ 824.1	1.011 2144	+2148	15 50.52	11 56 16.81	0.92
17	3 35 18.32		+19 17 39.3		1.011 4292		15 50.32	11 56 18.31	+ 1.50

R_C

FIGURE 22 – "AMERICAN EPHEMERIS"

6. CONCLUSION

The apparent place of α Tauri computed by both methods differ in X, Y, Z by

$$\delta X = 3 \times 10^{-8}$$

$$\delta Y = 2 \times 10^{-8}$$

$$\delta Z = 2 \times 10^{-8}$$

This results in an overall angular error of about 4×10^{-8} radians which is comparable to the accuracy (5×10^{-8} radians) of the star data in the "Apparent Place of Fundamental Stars."

Both methods are accurate. However, the method using mean place star data is preferred for two reasons:

1. This method requires about one-half the input data of that required for the method using apparent star place data.

2. This method uses star data that is already referred to the mean equator-equinox coordinate system of the nearest BNY, which is the desired system.

The apparent place computed using the mean place of α Tauri given in the "American Ephemeris" differs from that computed using the mean place given in the APFS in X, Y, Z by

$$\delta X = 0.55 \times 10^{-6}$$

$$\delta Y = 0.17 \times 10^{-6}$$

$$\delta Z = 0.15 \times 10^{-6}$$

This yields an overall angular error of about $.6 \times 10^{-6}$ radians which is within the accuracy (5×10^{-6} radians) of the star data in the A.E. Since the sextant, the primary optical navigation instrument for Apollo, can only be positioned to 10" (5×10^{-5} radians), the mean places of stars taken from the A.E. is sufficiently accurate.

7.0 ACKNOWLEDGEMENTS

I am indebted to W. G. Heffron who motivated me to apply vector and matrix mathematics to the subject of positional astronomy. The use of vectors and matrices allows, in my opinion, a clearer presentation of the physical phenomena and the coordinate transformations that determine a star's apparent place than the trigonometric expressions found in other books on positional astronomy.

I am indebted to Miss G. M. Cauwels who did all of the computer programming for this memorandum. She expertly and patiently did all the necessary debugging (required mostly because of errors in theory) and helped to correct some of the coordinate transformation mathematics used in the computer programs.

A. C. Brown, Jr.

2014-ACB-bjh

Attachments
Appendices A thru D

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APPENDIX A

RELATIVISTIC ABERRATION

Rigorously, the direct addition of velocity vectors used to determine the star's apparent place to a moving observer is incorrect. An exact treatment, based on relativity theory, is given in this appendix. From this exact equation for the star's apparent place, equations for both relativistic and non-relativistic aberration are found.

Consider a reference and observer that instantaneously occupy the same point in space. However, the observer is moving relative to the reference with a velocity of $\bar{V}$. Put two orthogonal coordinate systems, K and K', with their origins on the reference and the observer respectively. Since aberration is a physical phenomena, the orientation of coordinate systems does not matter. However, a good choice can simplify the mathematics. First, orient both K and K' so they are congruent to each other. Second, orient them so the velocity vector is along one of the axes (make it the x axis). And third, orient them so the reference direction to the star lies in one of the planes defined by the axes (make it the x-y plane). All of the above is shown in Figure 1.
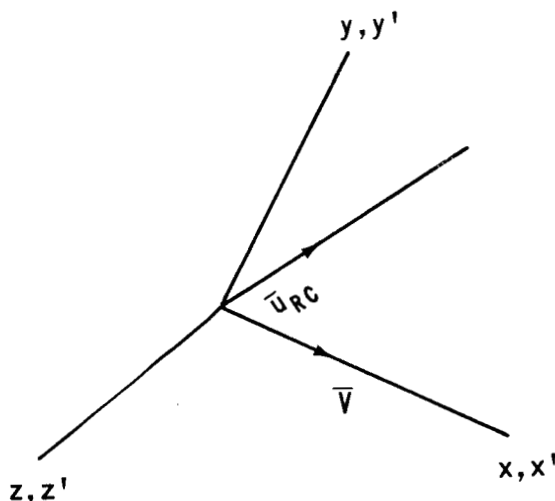

FIGURE I

Additionally, $\bar{u}_{RC}'$ is the unit apparent direction toward the star from the reference.

Appendix A (contd.)

The coordinate system of the observer (K') and the reference (K) can be related by the Lorentz transformations. They are as follows:

$$x' = \frac{x - Vt}{\sqrt{1 - \frac{V^2}{c^2}}} \tag{1}$$

$$y' = y \tag{2}$$

$$z' = z \tag{3}$$

$$t' = \frac{t - \frac{V}{c^2}x}{\sqrt{1 - \frac{V^2}{c^2}}} \tag{4}$$

Equations (1) through (4) are differentiated with respect to t'. Then,

$$v'_x = \frac{v_x - V}{\sqrt{1 - \frac{Vv_x}{c^2}}} \tag{5}$$

$$v'_y = \frac{v_y}{\sqrt{1 - \frac{Vv_x}{c^2}}} \tag{6}$$

and since $v_z = 0$,

$$v'_z = 0 \tag{7}$$
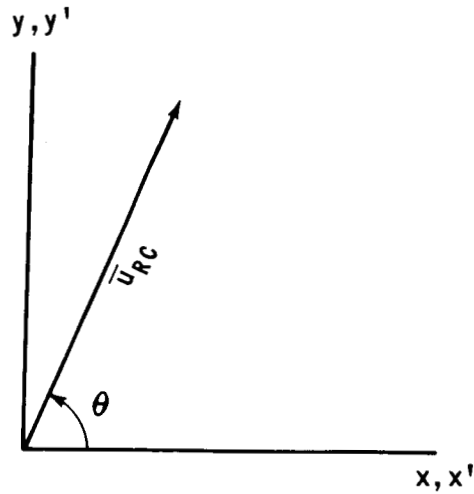
Appendix A (contd.)

FIGURE 2

In Figure 2, θ is the angle between the light ray and the x axis. The ray is directed toward the reference R. Thus

$$v_x = -c \cos \theta \qquad (8)$$

and

$$v_y = -c \sin \theta \qquad (9)$$

Substitution of equations (8) and (9) in (5) and (6), plus some manipulation, yield formulas for the direction of light to the moving observer.

$$\cos \theta' = \frac{\cos \theta + \dfrac{V}{c}}{1 + \dfrac{V}{c} \cos \theta} \qquad (10)$$

$$\sin \theta' = \frac{\sin \theta \sqrt{1 - \dfrac{V^2}{c^2}}}{1 + \dfrac{V}{c} \cos \theta} \qquad (11)$$

where θ', as shown in Figure 3, is the angle between the light ray toward the observer and the x axis.
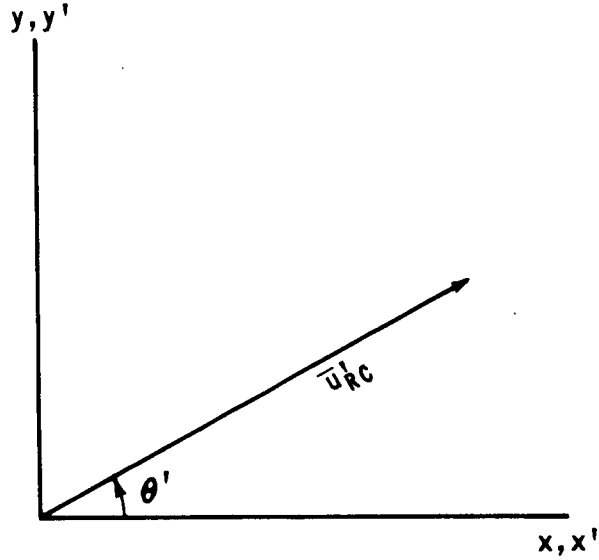
Appendix A (contd)

FIGURE 3

The result can be expressed vectorially by

$$\bar{u}'_{RC} \cdot \bar{u}'_x = \cos \theta'$$

$$\bar{u}'_{RC} \cdot \bar{u}'_y = \sin \theta'$$

where $\bar{u}'_{RC}$ is the unit apparent direction toward the star from the observer. The unit vector $\bar{u}'_{RC}$ can be expressed in components of the K system by

$$\bar{u}'_{RC} = (\bar{u}'_{RC} \cdot \bar{u}'_x) \, \bar{u}_x + (\bar{u}'_{RC} \cdot \bar{u}'_y) \, \bar{u}_y \tag{14}$$

and

$$\bar{u}'_{RC} = \frac{(\cos \theta + \frac{V}{c}) \, \bar{u}_x + \sqrt{1 - \frac{V^2}{c^2}} \, \sin \theta \, \bar{u}_y}{1 + \frac{V}{c} \cos \theta} \tag{15}$$

Appendix A (contd.)

Aberration ($\Delta\theta$) is equal to θ minus θ', and is obtained by vector multiplication of $\bar{u}_{RC}$ and $\bar{u}'_{RC}$

$$\sin \Delta\theta = \bar{u}_{RC} \times \bar{u}'_{RC} \qquad (16)$$

$$\sin \Delta\theta = \sin \theta \; \frac{\left[1 - \sqrt{1 - \frac{V^2}{c^2}} \right] \cos \theta + \frac{V}{c}}{1 + \frac{V}{c} \cos \theta} \qquad (17)$$

When $V \ll c$, one obtains the non-relativistic aberration formula

$$\sin \Delta\theta = \frac{V}{c} \sin \theta \qquad (18)$$

In the example of the earth orbiting the sun, the difference between relativistic aberration ($\Delta\theta$) and non-relativistic aberration ($\Delta\hat{\theta}$) is found. In this case, the aberration is small enough that

$$\Delta\theta = \sin \Delta\theta \qquad (19)$$

is valid.

The difference between the two types of aberration,

$$\delta = \Delta\theta - \Delta\hat{\theta} \qquad (20)$$

is obtained by subtracting equation (18) from equation (17)

$$\delta = \frac{\left[1 - \frac{V^2}{c^2} - \sqrt{1 - \frac{V^2}{c^2}} \right] \sin 2\theta}{2 \left(1 + \frac{V}{c} \cos \theta \right)} \qquad (21)$$

Appendix A (contd.)

By expanding the radical in equation (14) and neglecting all terms above the second power, one obtains

$$\delta = -\frac{.5\,\dfrac{V^2}{c^2}\sin 2\theta}{2\left(1 + \dfrac{V}{c}\cos\theta\right)} \tag{22}$$

The orbital velocity, V, of the earth is approximately 30 km/sec and the speed of light, c, is approximately 300,000 km/sec. Hence,

$$\frac{V}{c} = 10^{-4}$$

Consider the case where $\theta = 45°$ which maximizes the expression. Then substitute these into equation (22); one obtains

$$\delta = -\frac{.25 \times 10^{-8}}{1 + \dfrac{10^{-4}\sqrt{2}}{2}} \approx -.25 \times 10^{-8}$$

Since the mean places of stars given in the "Apparent Places of Fundamental Stars" is accurate to only 5×10^{-8}, relativistic aberration provides more accuracy than can be used.

APPENDIX B

THE E TERMS OF ABERRATION

The general practice of all mean place catalogs of stars is to include the so-called "E terms of aberration" in the star's mean place because these terms are relatively constant over long periods of time. This procedure reduces the computation required to obtain a star's apparent place from earth because velocity tables of the earth are not required. Generally, the E terms of aberration are described as the aberration caused by the eccentricity, the departure from circular motion, of the earth's orbit around the sun. As will be shown, the E terms of aberration are caused by one of two constant magnitude velocity vectors used to describe the earth's orbit. The constant magnitude vectors are derived first. Then, the aberration caused by each of these vectors are derived, and relevant comments on the E terms are given.

The ever-changing tangential velocity of one body orbiting another in an elliptical path can be expressed by two vector components in an orthogonal coordinate system. By a transformation, these vectors can be expressed by two constant magnitude vector components in a moving oblique coordinate system. The angle between these constant magnitude components accounts for the change in the magnitude and the direction of the orbital velocity vector.

It is desired to express the position and velocity in a plane orthogonal coordinate system defined by the earth's orbital plane. One axis is along its position vector $\bar{r}$; $\bar{u}_r$ is a unit vector along this axis. The other axis is perpendicular to $\bar{r}$ in the direction of increasing true anomaly f ; $\bar{u}_f$ is the unit vector along this axis. The position vector can be expressed by

$$\bar{r} = r\, \bar{u}_r \tag{1}$$

The tangential (or orbital) velocity is found by differentiating equation (1) with respect to time.

$$\bar{V}_T = \dot{r}\, \bar{u}_r + r\, \frac{df}{df}\, \bar{u}_f \qquad (2)$$

One can find the magnitude of this position vector from the equation of a conic

$$r = \frac{h^2\, \mu}{1 + e\, \cos\, (f - \bar{\omega})} \qquad (3)$$

where

h = angular momentum constant

μ = two body gravitational constant

e = eccentricity of the conic

$\bar{\omega}$ = argument of pericenter

Differentiating equation (3) with respect to time gives

$$\dot{r} = -\, \frac{h^2 e\, \sin\, (f - \bar{\omega})}{\mu[1 + e\, \cos\, (f - \bar{\omega})]^2}\, \frac{df}{df} \qquad (4)$$

and from the conservation of angular momentum,

$$r^2\, \frac{df}{dt} = h \qquad (5)$$

Substitution of equations (3), (4), and (5) into equation (2) and some manipulation yields

$$\bar{V}_T = \frac{\mu}{h}\, e\, \sin\, (f - \bar{\omega})\, \bar{u}_r + \frac{\mu}{h}\, [1 - e\, \cos\, (f - \bar{\omega})]\, \bar{u}_f \qquad (6)$$

for the tangential velocity.

Equation (6) can be written

$$V_T = V_r \, \overline{u}_r + V_f \, \overline{u}_f \tag{7}$$

where the velocity components V_r and V_f are respectively

$$V_r = - \frac{\mu}{h^2} \, e \sin(f-\overline{\omega}) \tag{8}$$

and

$$V_f = \frac{\mu}{h} [1-e \cos(f-\overline{\omega})] \tag{9}$$

One now transforms from the orthogonal polar coordinates to the moving oblique coordinates (see Mathematics of Circuit Analysis by Guillemin, p. 85-93) defined by two velocity vectors where one is perpendicular to the radius vector ($\overline{V}_f^*$) and the other ($\overline{V}_b^*$) is parallel to the semi-minor axis of the orbit ellipse. Thus, the velocity vector can be represented also by

$$\overline{V}_T = V_b^* \, \overline{u}_b^* + V_f^* \, \overline{u}_f^* \tag{10}$$
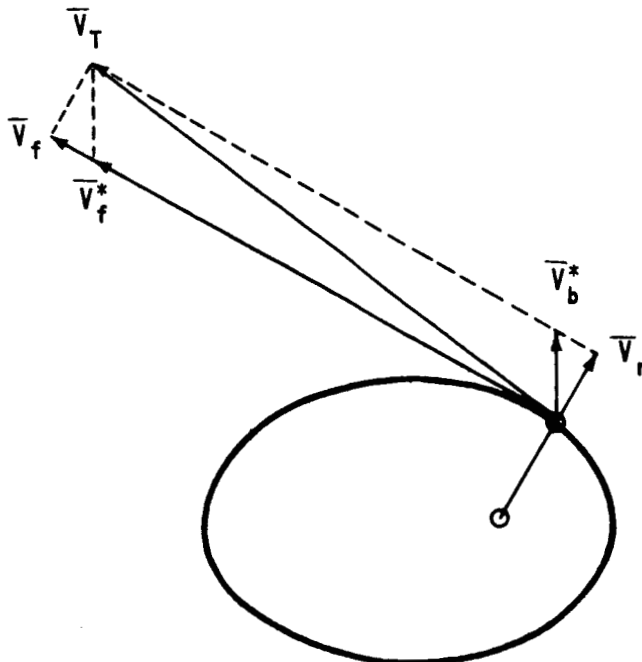
The vector relations are shown in Figure 1.

FIGURE 1

One expresses the orthogonal components in terms of the oblique components by

$$
\begin{bmatrix} V_r \\ V_f \end{bmatrix} = \begin{matrix} = \\ = \end{matrix} \begin{bmatrix} \sin (f-\bar{\omega}) & 0 \\ \cos (f-\bar{\omega}) & 1 \end{bmatrix} \begin{bmatrix} V_b^* \\ V_f^* \end{bmatrix} \tag{11}
$$

Then, take the inverse of the transformation matrix to solve for V_b^* and V_f^*,

$$
\begin{bmatrix} V_b^* \\ V_f^* \end{bmatrix} = \begin{matrix} = \\ = \end{matrix} \begin{bmatrix} \csc (f-\bar{\omega}) & 0 \\ -\cot (f-\bar{\omega}) & 1 \end{bmatrix} \begin{bmatrix} V_r \\ V_f \end{bmatrix} \tag{12}
$$

One obtains

$$
V_b^* = \frac{e\mu}{h} \tag{13}
$$

and

$$
V_f^* = \frac{\mu}{h} \tag{14}
$$

The aberration caused by each of these velocity vectors can be found using the equation (equation 12) for the apparent place due to aberration given in the text.

$$
\bar{u}_a = \text{unit}(\bar{u}_{RC} + \frac{\bar{V}}{c}) \tag{15}
$$

The reference ($\bar{u}_{RC}$) and the apparent ($\bar{u}_a$) direction are indicated in Figure 2.
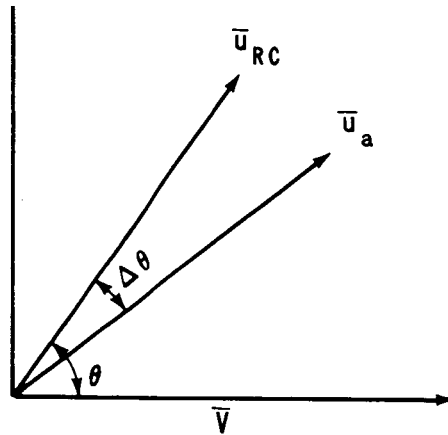
FIGURE 2

Drop the unitize symbol of equation (15). Then, take the vector product of $\bar{u}_R$ and the right side of equation 15 to obtain the aberration $\Delta\theta$ (shown in Figure 2).

$$\sin \Delta\theta = \bar{u}_{RC} \times \bar{u}_{RC} + \bar{u}_{RC} \times \frac{\bar{V}}{c} \qquad (16)$$

$$\sin \Delta\theta = \frac{V}{c} \sin \theta \qquad (17)$$

For small $\Delta\theta$,

$$\Delta\theta = \frac{V}{c} \sin \theta \qquad (18)$$

The aberration due to each velocity component can be found by substituting V_f^* and V_b^* for V in equation 18. The aberration caused by V_f^* is,

$$\Delta\theta = \frac{\mu}{ch} \sin \theta$$

and caused by V_b^* is,

$$\Delta\theta = \frac{\mu e}{ch} \sin\theta \qquad (20)$$

The earth's orbit is elliptical, so $\mu = (2\pi)^2/T^2 \, a^3$ and $h = (2\pi/T) \, a^2 \, (1-e^2)^{1/2}$. Substituting these quantities into the equations of aberration, one recognizes the factor

$$\frac{\mu}{ch} = \frac{2\pi \, a}{cT(1-e^2)^{1/2}} \qquad (21)$$

to be the formal definition (see "Text Book on Spherical Astronomy", by Smart, page 185) for the constant of aberration (κ). The constant is equal to 20''496 based on the value of a, the semi-major axis; of e, the eccentricity; of c, the speed of light; and of T, the period of rotation, adopted by the International Astronomical Union (IAU).

The aberration caused by V_f^* is,

$$\Delta\theta = \kappa \sin\theta \qquad (22)$$

and by V_b^* is,

$$\Delta\theta = e\kappa \sin\theta \qquad (23)$$

Aberration can also be expressed as a shift in longitude ($\Delta\lambda$) and latitude ($\Delta\beta$). In Figure 3, a star is located at X, but appears at X_1 because of aberration. Since stellar aberration is small, one considers the XX_1Y to be a plane triangle; ϕ is equal to the angle YXX_1 of this triangle

FIGURE 3

Then,

$$XY = \Delta\theta \; \cos\phi, \qquad YX_1 = \Delta\theta \; \sin\phi \qquad (24)$$

and

$$XY = -\Delta\beta, \qquad YX_1 = \Delta\lambda \; \cos\phi \qquad (25)$$

Again one solves for the aberration caused by each velocity vector. Set $V = V^*_f$, make appropriate substitutions, and solve for $\Delta\lambda$ and $\Delta\beta$.

$$\Delta\lambda = \kappa \sin \phi \sec \beta \qquad\qquad (26)$$

$$\Delta\beta = -\kappa \sin \theta \cos \phi \qquad\qquad (27)$$

Applying the sine formula to the sperical triangle, XSQ, yields

$$\sin \theta \sin \phi = \cos(\lambda_V - \lambda_*) \sin \beta \qquad\qquad (29)$$

where λ_V is the longitude of the velocity vector, and λ_* is the longitude of the star. Figure 4 shows the apparent orbit of the sun about the earth.
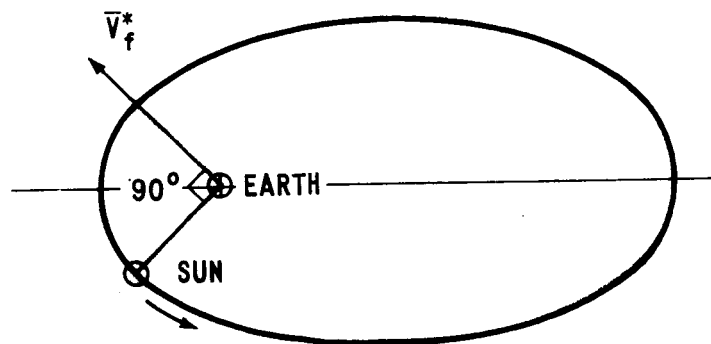

FIGURE 4

In the figure, the sun leads $\overline{V}_f^*$ by 90°. Thus,

$$\lambda_V = \lambda_\odot - 90° \qquad (30)$$

where $\lambda_\odot$ is the longitude of the sun. If all the substitutions are made, one obtains the final equation of aberration caused by $\overline{V}_f^*$,

$$\Delta\lambda = -\kappa \sec \beta \cos (\lambda_\odot - \lambda_*) \qquad (31)$$

and

$$\Delta\beta = -\kappa \sin \beta \sin (\lambda_\odot - \lambda_*) \qquad (32)$$

The aberration caused by $\overline{V}_b^*$ is obtained by using equation 23 for $\Delta\theta$ and letting λ_V be the longitude of the velocity vector, $\overline{V}_b^*$. Figure 5 shows the earth's orbit about the sun.
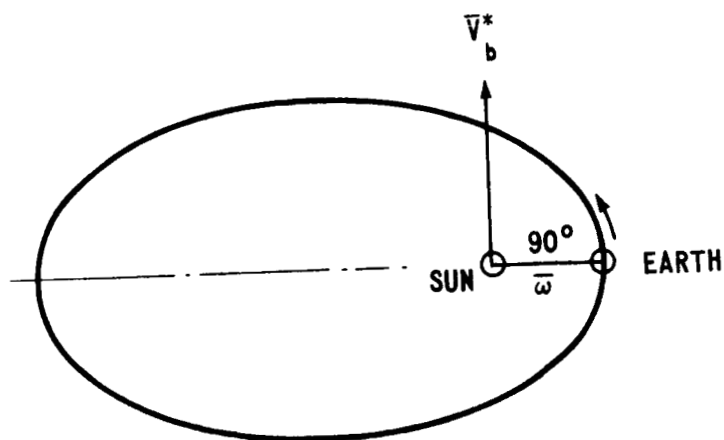

FIGURE 5

In the figure, $\overline{V}^*_b$ leads the argument of pericenter $\overline{\omega}$ by 90°. Thus

$$\lambda_V = \overline{\omega} + 90° \tag{33}$$

Therefore

$$\Delta\lambda = e\kappa \sec\beta \cos(\overline{\omega} - \lambda_*) \tag{34}$$

$$\Delta\beta = e\kappa \sin\beta \sin(\overline{\omega} - \lambda_*) \tag{35}$$

One recognizes equations (34) and (35) to be the formal definition of the so-called E-terms of aberration (see page 48 of the "Explanatory Supplement in the Ephemeris") which are included in the star's mean place given in all mean place catalogs of stars.

There are some important conclusions that can be based on these derivations. First, one notes that in this derivation there has been no mention of circular motion. Some might consider the aberration expressed by equations (31) and (32) as that due to mean circular motion since the velocity magnitude is constant and is based on the mean distance of the earth from the sun. However, recall that, in this derivation, an ellipse was used to find $\overline{V}^*_f$. The direction of $\overline{V}^*_f$ is based on the real position of the earth relative to the sun. This means that in any reduction from mean to apparent place, one uses the geometric longitude (or true anomaly) of earth and not the mean anomaly. Second, the E-term of aberration is that caused by the velocity component in the direction of the semi-minor axis; it is nothing more or less than that. In this sense, the E-terms of aberration do not include all the effects of the eccentricity of the earth's orbit about the sun.

APPENDIX C

NUTATION

The rigorous nutation matrix relating the true equator-equinox coordinates of date to the mean equator-equinox coordinates of date require the use of both the true obliquity (ϵ) and the mean obliquity (ϵ_o). This is because both coordinate systems are defined relative to the ecliptic system. Hence a transformation from one system of date to the other, must follow the sequence: True Equatorial-Ecliptic-Mean Equatorial. In the direction, true to mean, the transformation is given by the following three rotations. The first rotation is about the true equinox by an angle of ϵ; the second, about the ecliptic north pole by an angle of $\Delta\psi$; and the third, about the mean equinox by an angle of $-\epsilon_o$. The entire transformation matrix is:

		TRUE		
		X	Y	Z
MEAN	X_m	$c\Delta\psi$	$s\Delta\psi c\epsilon$	$s\Delta\psi s\epsilon$
	Y_m	$-s\Delta\psi c\epsilon_o$	$c\Delta\psi c\epsilon c\epsilon_o + s\epsilon s\epsilon_o$	$c\Delta\psi s\epsilon c\epsilon - c\epsilon s\epsilon_o$
	Z_m	$-s\Delta\psi s\epsilon_o$	$c\Delta\psi c\epsilon s\epsilon_o - s\epsilon c\epsilon_o$	$c\Delta\psi s\epsilon s\epsilon_o + c\epsilon c\epsilon_o$

$$(1)$$

where "s" and "c" stand for sine and cosine respectively.

Each of the terms in the nutation matrix could be represented by "a_{ij}" which denotes the term in the "i"th row and the "j"th column. By recognizing that

$$\cos \Delta\psi \approx 1 \qquad (2)$$

and by some manipulation, one obtains

$$a_{11} = a_{22} = a_{33} = 1 \qquad (3)$$

and

$$a_{32} = -\Delta\epsilon, \quad a_{23} = \Delta\epsilon \qquad (4)$$

where

$$\Delta\epsilon = \epsilon - \epsilon_o$$

Appendix C (cont'd)

The following relations are valid with negligible error:

$$\Delta\psi = \sin\Delta\psi$$

$$\Delta\epsilon = \sin\Delta\epsilon \qquad (5)$$

Then,

$$a_{21} = -\Delta\psi\cos\epsilon_o \, , \qquad a_{31} = -\Delta\psi\sin\epsilon_o \qquad (6)$$

and

$$a_{12} = \Delta\psi\cos\epsilon_o - \Delta\psi\Delta\epsilon\sin\epsilon_o \qquad (7)$$

Since the product of $\Delta\psi\Delta\epsilon$ is generally equal to 10^{-9} (within an order of magnitude), the second term of equation (7) provides more accuracy than can be used; the star places in the "Apparent Place of Fundamental Stars" are given only to 5×10^{-8}.

Hence,

$$a_{12} = \Delta\psi\cos\epsilon \qquad (8)$$

and, similarly,

$$a_{13} = \Delta\psi\sin\epsilon_o \qquad (9)$$

The final nutation matrix is as follows:

	X	Y	Z
X_m	1	$\Delta\psi\cos\epsilon_o$	$\Delta\epsilon\sin\epsilon_o$
Y_m	$-\Delta\psi\cos\epsilon_o$	1	$\Delta\epsilon$
Z_m	$-\Delta\psi\sin\epsilon_o$	$-\Delta\epsilon$	1

$$(10)$$

where one notes that the mean obliquity (ϵ_o) can be used throughout. By a similar argument, one could also show that the use of the true obliquity is equally valid.

APPENDIX D

Time of Greenwich Transit

A star's apparent place in the "Apparent Places of Fundamental Stars"(APFS) is given at the time the star transits the Greenwich meridian. The problem is to determine that time. The right ascension of a star uniquely determines the time of transit at any meridian. Obviously, stars with different positions (right ascension) along the equator cannot transit a meridian at the same time. Therefore, the time scale given in the APFS gives only the approximate time of transit.

A more accurate determination of the time of transit is based on the apparent place of the star given in the APFS, and the apparent sidereal time (A.S.T.). Because the star's apparent place is referred to the true equator-equinox coordinate system with the omission of the short period terms of nutation, this coordinate system is the reference system for the subsequent derivation. Specifically, all right ascensions are measured from the equinox defined by the intersection of the reference system and the ecliptic system at the approximate time of transit. Values of the A.S.T. at Greenwich, at 0^hU.T. and referred to the reference system are also tabulated in the APFS.

The time of transit is desired in universal time. Hence, one applies

$$U.T. = G.H.A.M.S. - 12^h \qquad (1)$$

where G.H.A.M.S. is the hour angle of the mean sun measured positive to the west from Greenwich along the true equator. The effect on the equatorial plane due to the short period terms of nutation is minor and can be ignored. Then, the G.H.A.M.S. can be related to the A.S.T. by

$$A.S.T. = G.H.A.M.S. + R.A.M.S. \qquad (2)$$

where R.A.M.S. is the right ascension of the mean sun. The right ascension of the star R.A.X. also can be related to the A.S.T. by

$$A.S.T. = G.H.A.X. + R.A.X. \qquad (3)$$

Appendix D (contd.)

where the G.H.A.X. is the hour angle of the star from the Greenwich meridian. Then, one equates the right hand sides of equation (2) and (3). This yields,

$$H.A.X. + R.A.X. = H.A.M.S. + R.A.M.S. \qquad (4)$$

Since the star transits at Greenwich,

$$G.H.A.X. = 0 \qquad (5)$$

Substituting equations (1) and (5) into (4) yields

$$R.A.X. = U.T. - 12^h + R.A.M.S. \qquad (6)$$

Since the A.S.T. is given for Greenwich and at 0^hU.T. in the APFS, G.H.A.M.S. = 12^h and

$$A.S.T. = 12^h + R.A.M.S. \qquad (7)$$

The time of transit is given by

$$U.T. = R.A. \ X - A.S.T._{0^h U.T.} \qquad (8)$$

in units of sidereal time. Equation (8) is represented pictorially in Figure 1, where G is the intersection of the Greenwich meridian with the equator. Υ is the equinox, and X is the star's position relative to the equator.
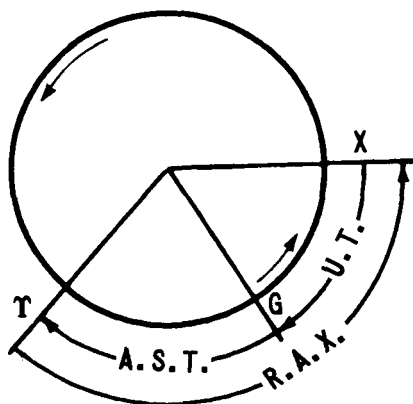

FIGURE I

Appendix D (con'd)

The positions of Υ and X change only a small amount during a day while G, a meridian, rotates with the earth. Since the A.S.T. is specified at 0^hU.T., the interval, G.X., determines the time the star will transit. This interval must be converted into units of the mean solar day. The conversion* is

$$1 \text{ sidereal day} = 0^d.9972697 \text{ mean days}$$

The time of transit in mean solar days is

$$\text{U.T.} = 0.9972697 \left(\text{R.A.X.} - \text{A.S.T.}_{0^h\text{U.T.}}\right) + d \qquad (9)$$

where R.A.X. and A.S.T. are in fractional parts of a sidereal day and d is the integer mean day corresponding to the tabulated A.S.T.

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*The scale of measure of GX is the true sidereal day with the omission of the short period terms of nutation. Conversion is defined only for mean S.T. to U.T. because of the variability of true S.T. measure caused by nutation. Rigorously, in this case, one converts from true S.T. to mean S.T. by applying the difference of the long period "equation of the equinoxes" over the interval of GX. This correction is quite small and therefore is ignored.